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DESIGN OF STABLE INTEGRATION SYSTEMS FOR MOTION MEASUREMENT

J. D. Gordon

Naval Ship Research and Development Center
Bethesda, Maryland

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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

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by

J. D. Gordon

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STRUCTURES DEPARTMENT
UNDERWATER EXPLOSIONS RESEARCH DIVISION
PORTSMOUTH, VIRGINIA

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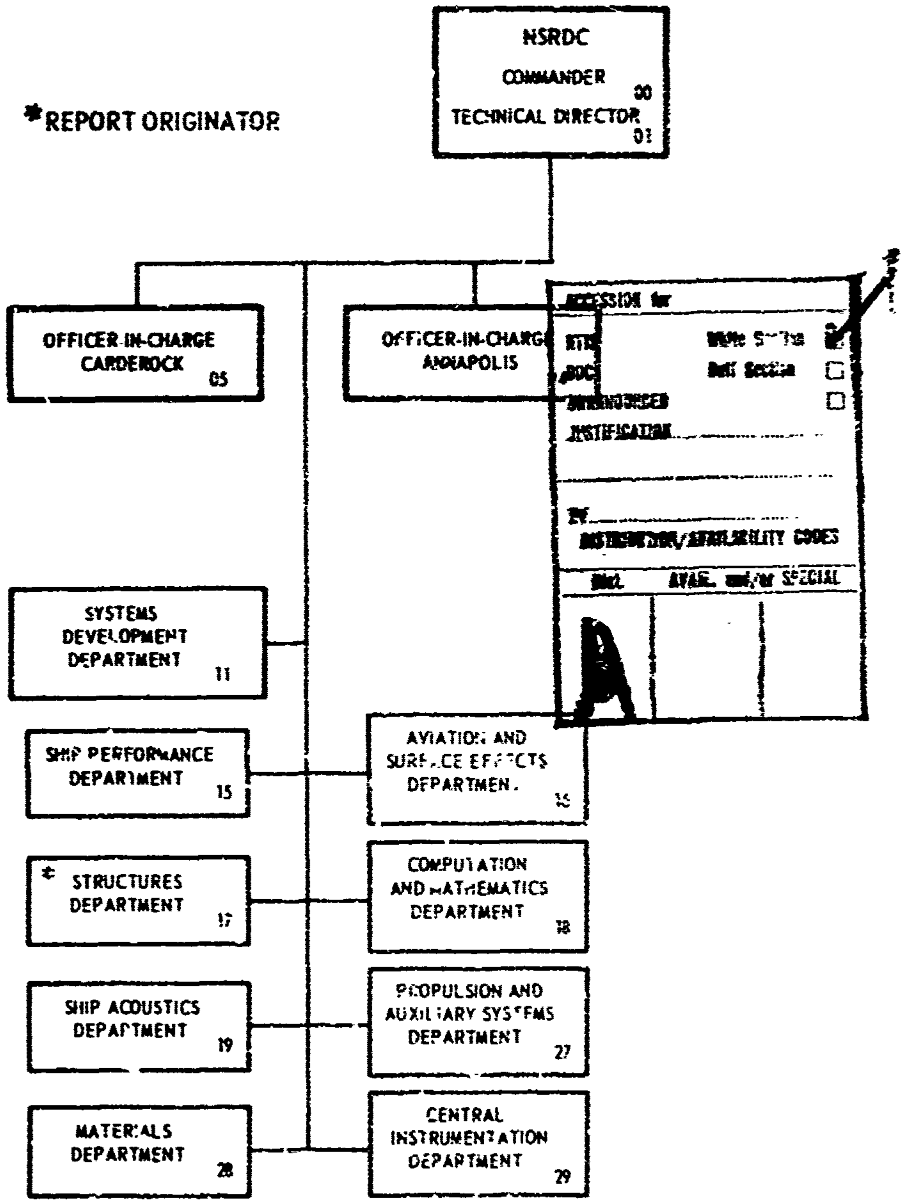

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ABSTRACT

Designing and testing damage-resistant Navy ships requires measurement of absolute ship motion caused by attacks from mines, depth charges, torpedoes, etc. This report gives a method of designing electronic integration systems, stabilized by feedback, which may be used to obtain absolute ship motion by integrating accelerometer outputs. Design data are given for several variations of the general integration method presented, and an example of each variation is demonstrated by obtaining displacement from acceleration by double integration.

ADMINISTRATIVE INFORMATION

The work presented in this report was begun under Defense Atomic Support Agency Project Number K-11BAXN, Task Area X501 and was completed under Navy Project Number S-F35.422.110, Task 15050.

INTRODUCTION

BACKGROUND

Navy ships and equipment must be designed to resist damage from underwater explosions due to mines, depth charges, torpedoes, etc. The design of damage-resistant ships and equipment which meet Navy shock and vibration specifications requires knowledge of the relationship between underwater explosions and damage. To obtain this knowledge, dynamic measurements of such mechanical parameters as displacement, velocity and acceleration must be made during underwater explosion experiments on ships. The severe shock environment in which sensing devices must operate and the lack of convenient stationary references for measurements of absolute motion during the experiment create unique instrumentation problems in underwater explosions work. This report deals with the problem of absolute motion measurement during explosion experiments and presents a solution to critical aspects of the problem.

During past underwater explosion tests, velocity and displacement measurements at various locations on the target ship have been made inertially by seismically correcting[1]

1. Walker, R.R., "A Procedure for the Correction of Velocity Meter Records," David Taylor Model Basin Report 1980 (Jun 1965).

A complete listing of references is given on page 51.

and integrating the output of velocity meters[2] and photographically by using fixed references. The limited displacement range of velocity meters and the problem of providing a stationary camera and reference for photographic measurement have shown a need for developing better methods for making these motion measurements.

Until recently accelerometers have not been available that have characteristics suitable for accurately sensing ship motions. However recent improvements in available accelerometers allow their use in many situations. These accelerometers provide a voltage output proportional to the sensed acceleration. Velocity and displacement may be obtained from the measured acceleration by integration and double integration, respectively. For the time duration required in underwater explosions work, the dynamic ranges of most general purpose instrumentation tape recorders are not large enough to record acceleration with the accuracy needed for accurate integration and double integration. Therefore the integration process used must operate directly on the signal generated. Since the accelerometer has a voltage output, it may be directly integrated using electronic integrators. Velocity and displacement initial values and the acceleration component due to gravity are unknown because target motion prior to the test is present. Therefore open loop operation of the integration system, which requires starting the integrators with correct initial values immediately before the test, is not practical. A practical integration system must be capable of being started sometime ahead of the test so as to automatically determine velocity and displacement initial values at the beginning of the transient acceleration integrated.

OBJECTIVE

The purpose of this report is to present and demonstrate a practicable method of performing multiple integration of a transducer output signal that will be applicable under field-test conditions. To meet this objective, design data are given for several variations of the general integration method presented, and an example of each variation is demonstrated by obtaining displacement from acceleration by double integration.

2. Gordon, J.D., "Analysis and High Frequency Correction of the Bar-Magnet Velocity Meter Response," David Taylor Model Basin Report 2187 (Apr 1966).

A complete listing of references is given on page 51.

APPROACH

Initial value and startup problems will be surmounted by providing the integration system with feedback so that steady-state motion will be reproduced at the integration system output with the loss of very low frequency components only. The feedback arrangement and parameters will be chosen so that transient motion due to the test will be accurately reproduced for a finite time after the beginning of the transient. The steady-state requirements necessitate analysis of the integration system from the viewpoint of stability, and the transient requirements dictate the nature of the feedback used to stabilize the integration system. If the stabilizing signal being fed back is the integrator output delayed in time, the integral of a transient signal will be exact for a time interval measured from the beginning of the transient and of duration equal to that of the feedback delay. After the duration of the delay, an exaggerated error is produced in the integrated result as the output signal goes on to meet the requirements of stability.

The feedback delay may be obtained in either of two ways or through a combination of both. The first is to interpose between the output and the feedback a network having capacitors which must be charged by the output before the output signal may be fed back. Charging the capacitors requires time, resulting in a delay effect characterized by a gradual increase in integral error. The second approach utilizes a tape-recorder transport delay. As field-test results are usually recorded on magnetic tape, an efficient use of equipment would be to use the tape recorder transport delay between the record and reproduce heads to provide the feedback delay. This second method of obtaining feedback delay introduces error only after the duration of the delay. A combination of tape transport delay and capacitor network delay results in exact integration for the duration of the tape delay, followed by a gradual deterioration in integral accuracy.

The ideas developed thus far are qualitative. Laplace transform transfer-function techniques are used to derive numerical values for all parameters in the design of the stable integration system. The basic design technique is to determine all design parameters so that a transfer function appearing in the mathematical analysis will have the overall characteristics of a unit delay. This transfer function is called the unit-delay approximation and contains the transfer functions of both the network delay and the tape delay as well as the integrator feedback parameters.

Through an investigation of a physical system described by a stable transfer function possessing a zero at the origin, it is shown that stable multiple integration may be obtained, having any desired degree of accuracy for a finite time. Several integration system designs differing in the nature of their error will be presented. Two system designs exhibiting unidirectional error are fully treated; whereas, a third system design exhibiting ripple error is discussed, and an example is given. The systems are investigated, using an analog computer, and the practicability of the integration method is then demonstrated by using it to obtain displacement from an accelerometer. The double integration involved is understood to take place simultaneously with the generation of the acceleration.

THE STABILIZING TRANSFER FUNCTION

PROBLEM OF INSTABILITY

The transfer function of an initially quiescent linear system is defined as the ratio of the Laplace transform of the output to the Laplace transform of the input. The system is stable if the output due to a finite-step input remains finite for all time and is unstable if the output increases without limit.

The transfer function of the system accomplishing multiple integration is

$$\frac{E_1(s)}{E_2(s)} = \frac{1}{s^i} \tag{1}$$

where $E_2(s)$ is the transformed signal to be integrated, and $E_1(s)$ is the transformed result of i integrations. The system described by Equation (1) is unstable. This instability is the basic problem with integration.

Let the transfer function

$$\frac{E_0(s)}{E_1(s)} = F(s) \tag{2}$$

possess a zero of order i or greater at the origin of the s-plane. With suitable selection of F(s), the problem of instability may be eliminated without significantly affecting the result of the integration of a transient by constructing an integration system which possesses an overall transfer function. The transfer function is the product of corresponding sides of Equations (1) and (2). Thus $E_1(s)$, the transformed result of exact integration,

is eliminated. This elimination results in

$$\frac{E_o(s)}{E_2(s)} = \frac{1}{s^j} F(s) \tag{3}$$

where $E_2(s)$ is the transformed signal to be integrated, and $E_o(s)$ is the transformed output. The system described by Equation (3) is stable because the zero at the origin inherent in $F(s)$ cancels the pole at the origin contributed by $\frac{1}{s^j}$. Therefore $F(s)$ is called the stabilizing transfer function.

Equation (3) is not to be interpreted as describing two distinct systems in cascade. Equation (3) is the transfer function of one system having overall characteristics of the two systems described by Equations (1) and (2) as if they were in cascade. The system described by Equation (3) may be regarded as performing the integration described by Equation (1) and then passing the result through the system described by Equation (2). However, these two operations are performed simultaneously, not individually. The form and parameters of $F(s)$ will be determined to meet the requirement that transients be integrated as accurately as possible.

FORM AND PROPERTIES OF THE STABILIZING TRANSFER FUNCTION

Three closed-loop systems, each consisting of one or more integrators with feedback; an adder; a tape-transport delay; and a network delay having poles only are shown in Figure 1. The number of integrators with feedback shown for each system is j, and the differential order of the system is k. The transfer function of the tape-transport delay is $\exp(-\theta s)$, and the transfer function of the network delay is

$$G(s) = \frac{J_j}{\sum_{n=j}^{k} J_n s^{n-j}}$$

For each system shown in Figure 1, the transfer function between the input and output of the adder is given by

$$\frac{E_o(s)}{E_1(s)} = F(s) = \frac{\sum_{n=j}^{k} J_n s^n}{\exp(-\theta s) \sum_{n=o}^{j-1} J_n s^n + \sum_{n=j}^{k} J_n s^n} \tag{4}$$

where $J_k = 1$,

θ is the delay time of the tape transport, and

J_n are coefficients to be determined.

Equation (4) is the stabilizing transfer function. The numerator of Equation (4) has a zero of order j at the origin of the s-plane which cancels the s^j in the denominator of Equation (3) for values of i through j.

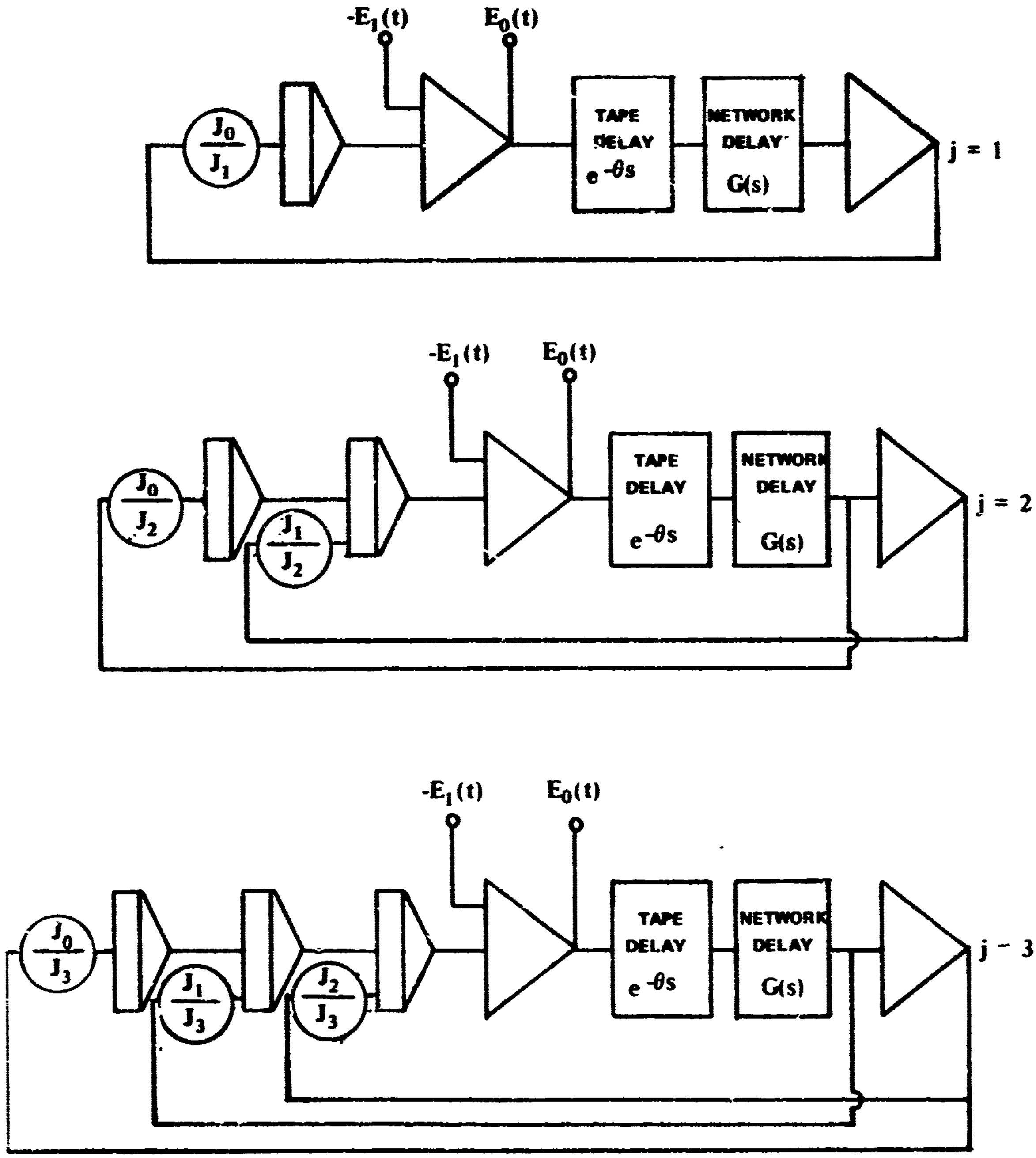

Figure 1 - The Analog Simulation of the Stabilizing Transfer Function

Assuming that the system described by Equation (4) is stable, application of the final value theorem of the Laplace transform to the transformed step response of the system described by Equation (4) shows the characteristics of the step response which are independent of the values of the coefficients in Equation (4). Application of this theorem shows that the step response and its integrals through the $(j-1)^{th}$ integral approach zero as time grows infinite. In order for the integral conditions to hold, the step response must have appropriate amounts of positive and negative area. These areas are produced by an oscillating response. The minimum number of oscillations possible in the step response is that necessary to produce the minimum number of positive and negative areas so that the integral conditions just described will hold. The minimum number of oscillations is therefore determined by the order of the zero at the origin of the describing transfer function.

When the number of oscillations present in the step response is equal to the minimum determined by the value of j, the system will be called very stable. When the number of oscillations is greater than the minimum but finite in amplitude, the system will be called oscillatory. Since unavoidable oscillations occur in the latter time response to an input step, the step may not be reproduced at these latter times, and attention must be focused on step reproduction in the interval of time immediately after the occurrence of the step.

COEFFICIENT DETERMINATION

Equation (4) may be given in the form

$$\frac{E_o(s)}{E_i(s)} = 1 - \sum_{n=o}^{j-1} \frac{J_n}{J_o} s^n \frac{J_o}{\sum_{n=o}^{j-1} J_n s^n + \exp(\theta s) \sum_{n=j}^{k} J_n s^n} \tag{5}$$

From Equation (5), it is seen that the output of the system described by Equation (4) is equal to its input minus the responses of the input and its first j-1 consecutive derivatives (each weighted by $\frac{J_n}{J_o}$) to the system described by the transfer function

$$\frac{E_3(s)}{E_4(s)} = \frac{J_o}{\sum_{n=o}^{j-1} J_n s^n + \exp(\theta s) \sum_{n=j}^{k} J_n s^n} \tag{6}$$

If the response of the system described by Equation (6) to the input of the system described by Equation (5) and its j-1 consecutive derivatives is delayed in time, the output of the system described by Equation (5) will be equal to its input for the duration of the delay. Therefore the J coefficients will be determined by a criterion of approximation, which gives the system described by Equation (6) the essential character of a unit delay. A variable delay may be obtained by time scaling.

Equation (6) will be called the unit-delay approximation. It contains the tape and network-delay transfer functions as well as the integrator feedback parameters. When all quantities in Equation (6) are specified, the stabilizing transfer function is completely determined. The criterion for unit-delay approximation will now be established.

After some manipulations with the MacLaurin series of the natural log of Equation (6), Equation (6) becomes

$$\frac{E_3(s)}{E_4(s)} = \exp\left(-\sum_{m=1}^{\infty} C_{2m} s^{2m}\right) \exp\left(-\sum_{m=1}^{\infty} C_{2m-1} s^{2m-1}\right) \tag{7}$$

where the C coefficients are functions of the J coefficients and θ. By making C_1 unity and the next k-1 consecutive odd-subscripted coefficients zero, k J coefficients are determined, and Equation (7) may be approximated by

$$\frac{E_3(s)}{E_4(s)} \approx \left[\exp\left(-\sum_{m=1}^{k} C_{2m} s^{2m}\right)\right] \exp(-s) \tag{8}$$

Since exp(-s) is the transform of a unit delay, the unit impulse response of the system approximated by Equation (8) is the inverse transform of the function of s in the brackets, shifted to the right one unit of time.

It can be shown that the double sided inverse transform of

$$\exp\left(-\sum_{m=1}^{k} C_{2m} s^{2m}\right) \text{ is } D_o \exp\left(-\sum_{m=1}^{\infty} D_{2m} t^{2m}\right) \tag{9}$$

where D_o is chosen so that the area under the time function is unity, and the remaining D coefficients are determined from the C coefficients. Expression (9) is similar to the standard normal curve. Therefore the unit-impulse response of the system approximated by Equation (8) may be regarded as a normal type of curve, centered about unit time. As the order of the unit-delay approximation increases, the normal type of curve becomes higher and narrower, approaching a unit impulse delayed one unit of time. Thus it is seen that use of the criterion selected to determine the J coefficients gives the system described by Equation (6) unit-delay properties.

The J coefficients are readily evaluated in terms of n, θ, and k if a set of linear equations is solved which guarantees that conditions put on the coefficients of the odd powers of s in Equation (7) are met. The derivation of this set of linear equations follows.

Let the MacLaurin expansion of the reciprocal of Equation (6) be given by

$$\sum_{n=o}^{j-1} \frac{J_n}{J_o} s^n + \exp(\theta s) \sum_{n=j}^{k} \frac{J_n}{J_o} s^n = \sum_{n=o}^{\infty} B_n s^n \tag{10}$$

Combining Equations (10) and (7) yields

$$\sum_{n=o}^{\infty} B_n s^n = \exp\left(\sum_{n=1}^{\infty} C_n s^n\right) \tag{11}$$

where $B_o = 1$. It may be shown that when $C_1 = 1$ and the next k-1 odd-subscripted C coefficients are zero,

$$1 = B_1$$

$$\frac{1}{3} = B_2 - B_3$$

$$\frac{1}{15} = B_3 - 3B_4 + 3B_5$$

$$\vdots$$

$$\frac{1}{A_{o,n}} = \sum_{u=n}^{2n-1} (-1)^{u-n} A_{2n-1-u,\, n-1} B_u$$

$$n = 1, 2, \ldots, k \tag{12}$$

where the A coefficients are seen to be the Bessel Polynomial coefficients[3] and are given by

$$A_{\alpha,\beta} = \frac{2^{\alpha}(2\beta-\alpha)!}{2^{\beta}\alpha!\,(\beta-\alpha)!}$$

Reference 3 - Krall, H.L. and Orrin Frink, "A New Class of Orthogonal Polynomials: The Bessel Polynomials," Transactions of the American Mathematical Society, Vol. 65, No. 1, pp. 100-107 (Jan 1949).

A complete list of references is given on page 51.

The B coefficients may be given in terms of the J coefficients and θ by

$$\left.\begin{aligned} B_u &= \frac{J_u}{J_0}, \quad 0 \leq u \leq j-1 \\ B_u &= \sum_{v=j}^{k} \frac{\theta^{u-v}}{(u-v)!} \frac{J_v}{J_0}, \quad u \geq v \end{aligned}\right\} \qquad (13)$$

Substitution of Equations (13) in Equations (12) gives

$$\left.\begin{aligned} \frac{1}{A_{0,n}} &= \sum_{u=n}^{2n-1} (-1)^{u-n} A_{2n-1-u,\,n-1} \frac{J_u}{J_0}, \quad j \leq 3 \\ & \qquad n = 1, \ldots, j-1 \\ \frac{1}{A_{0,n}} &= \sum_{v=j}^{k} \sum_{u=n}^{2n-1} (-1)^{u-n} A_{2n-1-u,\,n-1} \frac{\theta^{u-v}}{(u-v)!} \frac{J_v}{J_0} \\ & \qquad n = j, \ldots, k \qquad u \geq v \end{aligned}\right\} \qquad (14)$$

The expansion of Equations (14) gives k linear simultaneous equations in k unknown J coefficient ratios. After specifying a particular value of θ, these equations may be solved for the J coefficients corresponding to that value of θ.

MAC LAURIN SYSTEM

A system designed from a unit-delay approximation having zero θ will be called a MacLaurin system. Solution of Equations (14) for zero gives the J coefficients for MacLaurin systems directly. They are

$$J_n = A_{n,k} = \frac{2^n (2k-n)!}{2^k n! (k-n)!} \qquad (15)$$

Table 1 lists the J coefficients for MacLaurin systems with k ranging from 1 to 10.

TAPE-DELAY SYSTEM

A system designed from a unit-delay approximation having θ greater than zero will be called a tape-delay system. The solution of Equations (14) with θ varied shows that if (k-j) is odd, J_k/J_0 passes through zero, going negative as θ is increased. The system is very stable until instability occurs with the change in sign of J_k/J_0. Therefore θ may not be further increased. For the value of k and θ producing zero J_k/J_0, the

remaining J coefficient ratios are equal to the J coefficient ratios calculated for the next lower order and the same value of θ. Thus two adjacent orders give the same unit delay approximation for a certain value of θ. As θ is increased from this value, the lower of the two orders becomes oscillatory. Therefore the value of θ producing zero J_k/J_0 for (k-j) odd is the optimum value of θ. For θ optimum Table 2 lists θ and the corresponding J coefficients for the first three values of j with k ranging from j to 10.

STEP RESPONSES

Now that the J coefficients have been determined, and the stabilizing transfer function of Equations (4) and (5) is completely defined, the characteristics of the stabilizing transfer function will be studied through analog simulation.

Figure 1 shows the analog simulation of the stabilizing transfer function for the first three values of j. By recording the output of the adder, or magnetic tape, and by simultaneously playing the output back into the input of the simulation of G(s), a signal delay equal to the time required for the tape to pass from the record head to the reproduce head will be obtained. Further delay in the MacLaurin sense is obtained with the simulation of G(s). For convenience in simulation, G(s) may be factored and given in the form

$$G(s)= \frac{L}{s+L} \prod_{n=1}^{\left[\frac{k-j}{2}\right]} \frac{\omega_n^2}{s^2+2\zeta_n\omega_n s+\omega_n^2} \tag{16}$$

where the brackets indicate fractions are dropped. Table 3 gives the parameters of Equation (16) for the MacLaurin systems of Table 1 for the first three values of j. Table 4 gives the parameters of Equation (16) for the tape-delay systems of Table 2. Figures 2 through 4 are step responses of the MacLaurin system obtained by analog simulation for j = 1,2, and 3, respectively, with k ranging from j to 10. Figures 5 through 7 are the initial portions of Figures 2 through 4, respectively, with scales expanded. Figures 8 through 10 are step responses, obtained by analog simulation of the tape-delay systems given in Table 2 for j=1, 2, and 3, respectively. Figures 11 through 13 are the initial portions of Figures 8 through 10 with scales expanded.

TABLE 1 - COEFFICIENTS FOR MAC LAURIN SYSTEMS

k	1	2	3	4	5	6	7	8	9	10
J_0	1	3	15	105	945	10395	1351[illegible]5	2027025	34459425	654729075
J_1	1	3	15	105	945	10395	135135	2027025	34459425	654729075
J_2		1	6	45	420	4725	62370	945945	16216200	310134825
J_3			1	10	105	1260	17325	270270	4729725	91891800
J_4				1	15	210	3150	51975	945945	18918900
J_5					1	21	378	6930	135135	2837835
J_6						1	28	630	13860	315315
J_7							1	36	990	25740
J_8								1	45	1485
J_9									1	55
J_{10}										1

TABLE 2 - COEFFICIENTS FOR TAPE-DELAY SYSTEMS

	j=1				
k	1	3	5	7	9
θ	0.4226	0.3708	0.3500	0.3385	0.3311
J_0	1	34.1277360	4510.81781	1329260.84	693322028.
J_1	1	34.1277360	4510.81781	1329260.84	693322028.
J_2		3.28592458	581.202307	193869.819	108472945.
J_3		1	176.674207	58967.8839	33005654.2
J_4			11.8162242	5614.63968	3565754.72
J_5			1	573.406028	406502.580
J_6				25.8561622	26152.7020
J_7				1	1462.31090
J_8					45.4951011
J_9					1

TABLE 2 - (Continued)

	j=2				
k	2	4	6	8	10
θ	0.2254	0.2447	0.2547	0.2610	0.2653
J_0	2.32380000	148.916012	28255.8611	10695759.2	6755471750.
J_1	2.32380000	148.916012	28255.8611	10695759.2	6755471750.
J_2	1	69.2787189	13482.1256	5166763.19	3287049860.
J_3		2.69754549	629.607853	252984.921	163171620.
J_4		1	265.513663	114820.092	77841489.9
J_5			11.4122236	6789.86134	5231723.54
J_6			1	734.952459	623565.406
J_7				25.9013965	31756.6879
J_8				1	1724.86212
J_9					46.1199401
J_{10}					1

TABLE 2 - (Continued)

	j=3			
k	3	5	7	9
θ	0.1389	0.1744	0.1954	0.2096
J_0	9.18359722	974.670874	252324.053	119280618.
J_1	9.18359722	974.670874	252324.053	119280618.
J_2	4.06119906	455.134786	120434.030	57595338.4
J_3	1	130.244495	36328.0132	17835132.3
J_4		2.44858972	626.586974	2890702.275
J_5		1	407.300308	233072.471
J_6			11.8795780	8905.14401
J_7			1	937.714614
J_8				27.3196023
J_9				1

TABLE 3 - NETWORK DELAY PARAMETERS FOR MAC LAURIN SYSTEMS

j	k	L	n	ζ_n	ω_n
1	2	3.00000			
1	3	∞	1	0.77459	3.87298
1	4	5.23548	1	0.53195	4.47833
1	5	∞	1	0.90889	6.20638
			2	0.37533	4.95309
1	6	7.47150	1	0.75293	6.99320
			2	0.28101	5.33374
1	7	∞	1	0.95157	8.49114
			2	0.22662	5.62469
			3	0.60353	7.69695
1	8	9.71062	1	0.85382	9.36765
			2	0.19545	5.82274
			3	0.47852	8.37620
1	9	∞	1	0.97008	10.75576
			2	0.17485	5.94309
			3	0.74550	10.15438
			4	0.38222	9.04371
1	10	11.95128	1	0.90399	11.68398
			2	0.15848	6.01739
			3	0.64199	10.88054
			4	0.31245	9.67548
1	11	∞	1	0.97971	13.01182
			2	0.14466	6.06790
			3	0.82407	12.52995
			4	0.54853	11.56306
			5	0.26397	10.25041

TABLE 3 - (Continued)

j	k	L	n	ζ_n	ω_n
2	3	6.00000			
2	4	∞	1	0.74535	4.70829
2	5	8.45663	1	0.40424	7.04735
2	6	∞	1	0.90217	9.42144
			2	0.27415	7.29597
2	7	10.85906	1	0.73660	10.11039
			2	0.14983	7.49589
2	8	∞	1	0.94892	11.89864
			2	0.06739	7.65384
			3	0.57991	10.67960
2	9	13.23045	1	0.84638	12.73167
			2	0.01137	7.77140
			3	0.44868	11.18927
2	10	∞	1	0.96873	14.30217
			2	-0.02807	7.85238
			3	0.73319	13.44594
			4	0.34558	11.66216
2	11	15.58141	1	0.89990	15.21222
			2	-0.05716	7.90414
			3	0.62528	14.08678
			4	0.26137	12.10229
2	12	∞	1	0.97892	16.66992
			2	-0.07961	7.93525
			3	0.81690	16.01552
			4	0.19806	12.50480
			5	0.52815	14.67993

TABLE 3 - (Continued)

j	k	L	n	ζ_n	ω_n
3	4	10.00000			
3	5	∞	1	0.73192	10.24695
3	6	12.25375	1	0.43126	10.14029
3	7	∞	1	0.89864	13.07732
			2	0.22335	10.06509
3	8	14.61088	1	0.72696	13.55985
			2	0.08347	10.03011
3	9	∞	1	0.94738	15.59357
			2	−0.01260	10.01610
			3	0.56398	13.92430
3	10	16.98404	1	0.84183	16.31838
			2	−0.08048	10.00838
			3	0.42662	14.24218
3	11	∞	1	0.96791	18.03169
			2	−0.12984	9.99882
			3	0.72527	16.90923
			4	0.31514	14.53723
3	12	19.35499	1	0.89732	18.87961
			2	−0.16684	9.98431
			3	0.61404	17.42477
			4	0.22595	14.81509
3	13	∞	1	0.97842	20.43492
			2	−0.19541	9.96481
			3	0.31221	19.60343
			4	0.15497	15.07403
			5	0.51342	17.89509

TABLE 4 - NETWORK DELAY PARAMETERS FOR TAPE-DELAY SYSTEMS

j	k	θ	n	ζ_n	ω_n
1	3	0.37080	1	0.28123	5.84189
1	5	0.35000	1	0.14074	6.06491
			2	0.45643	11.07396
1	7	0.33850	1	0.09982	6.18050
			2	0.24384	11.33789
			3	0.58021	16.45313
1	9	0.33110	1	0.07882	6.22178
			2	0.16293	11.88626
			3	0.35320	16.09377
			4	0.66156	22.12328
2	4	0.24470	1	0.16144	8.32338
2	6	0.25470	1	-0.04430	8.01464
			2	0.41837	14.48754
2	8	0.26100	1	-0.12089	7.98908
			2	0.17171	13.80260
			3	0.56014	20.61346
2	10	0.26530	1	-0.15783	7.96908
			2	0.05266	13.94195
			3	0.32091	19.21329
			4	0.64852	26.85772
3	5	0.17440	1	0.10727	11.41247
3	7	0.19540	1	-0.13246	10.32655
			2	0.39593	18.45666
3	9	0.20950	1	-0.23085	10.01881
			2	0.13032	16.72688
			3	0.54732	25.20036

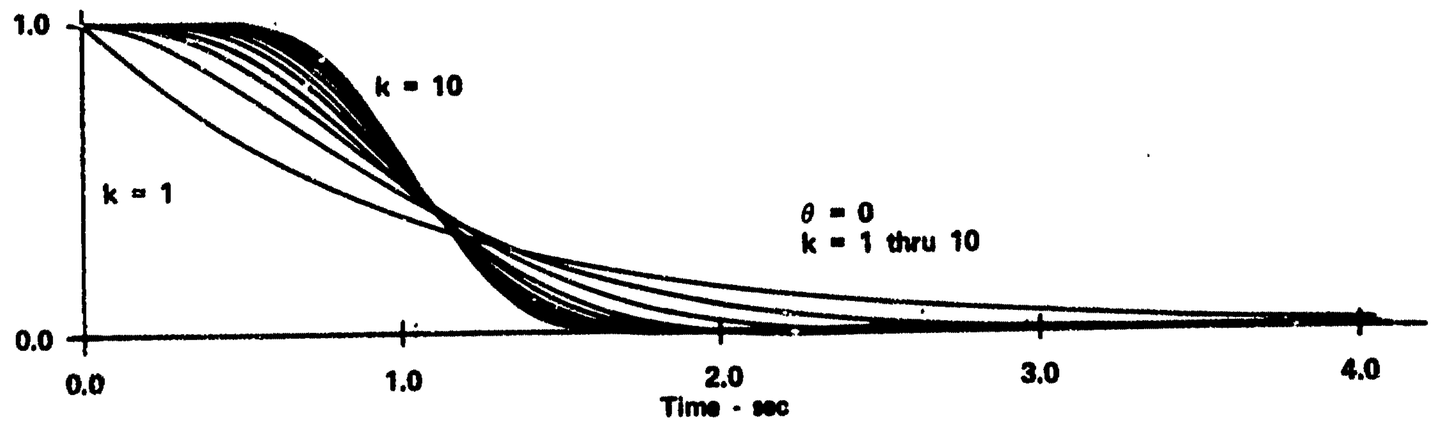

Figure 2 - Step Responses of MacLaurin Systems with j=1

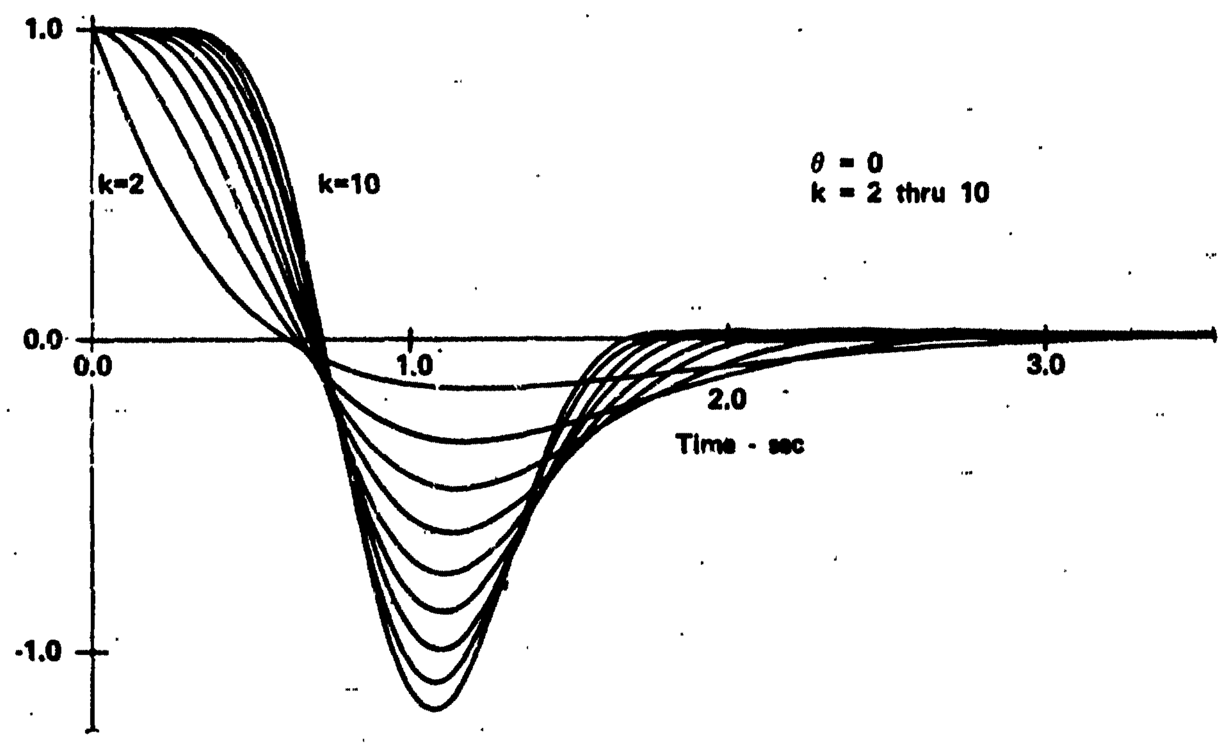

Figure 3 - Step Responses of MacLaurin Systems with j=2

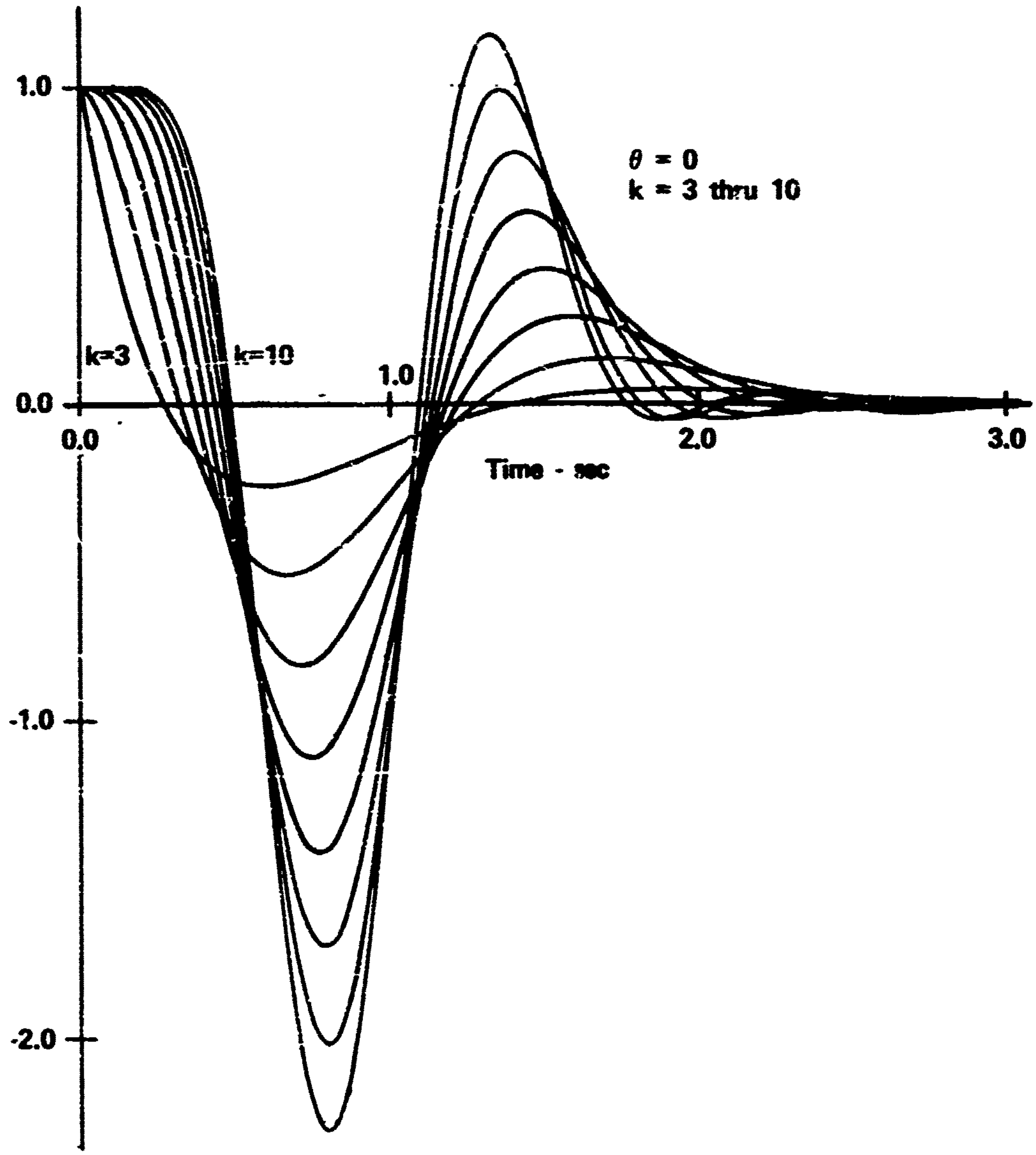

Figure 4 - Step Responses of MacLaurin Systems with j=3

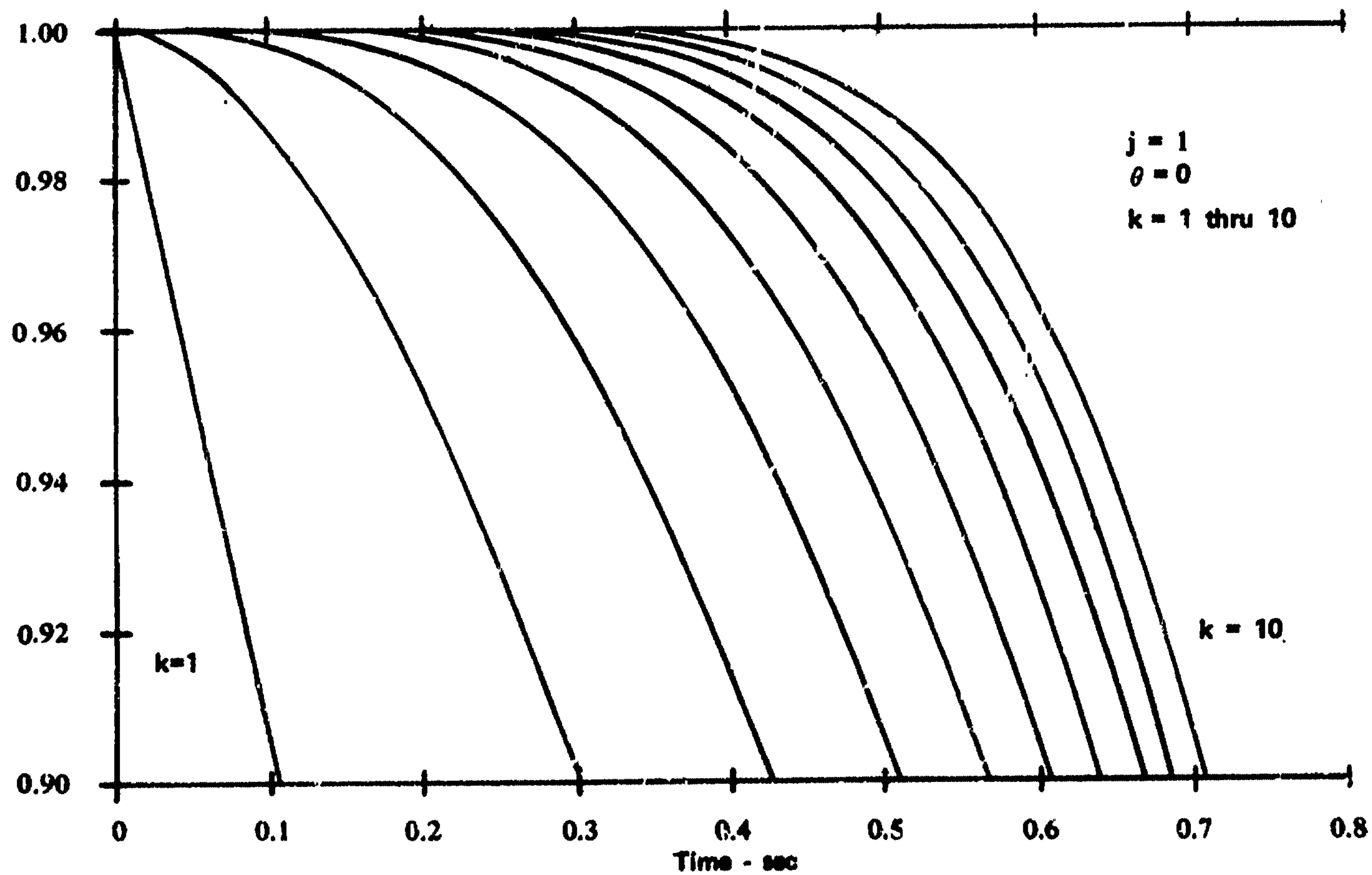

Figure 5 - Figure 2 with Scales Expanded

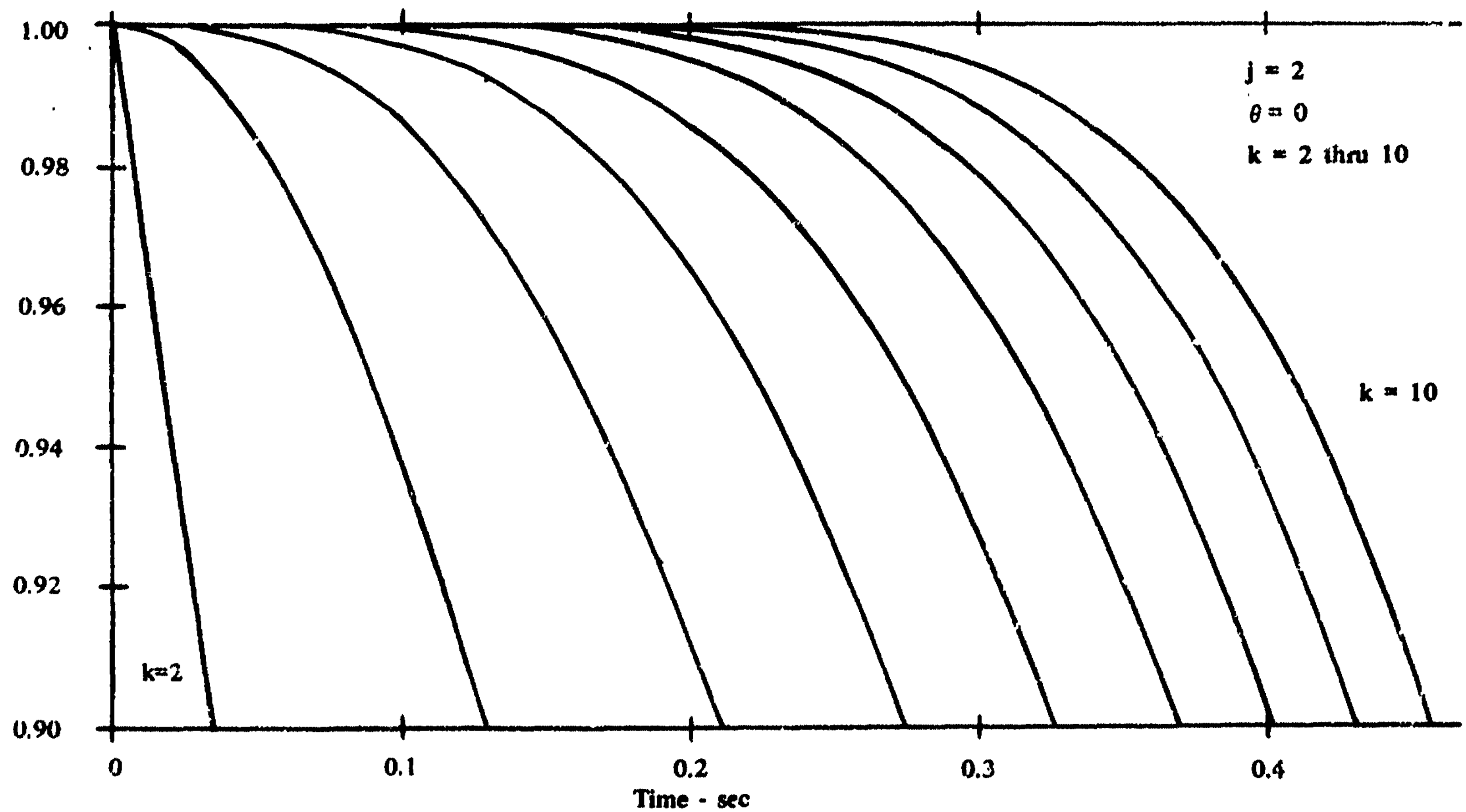

Figure 6 - Figure 3 with Scales Expanded

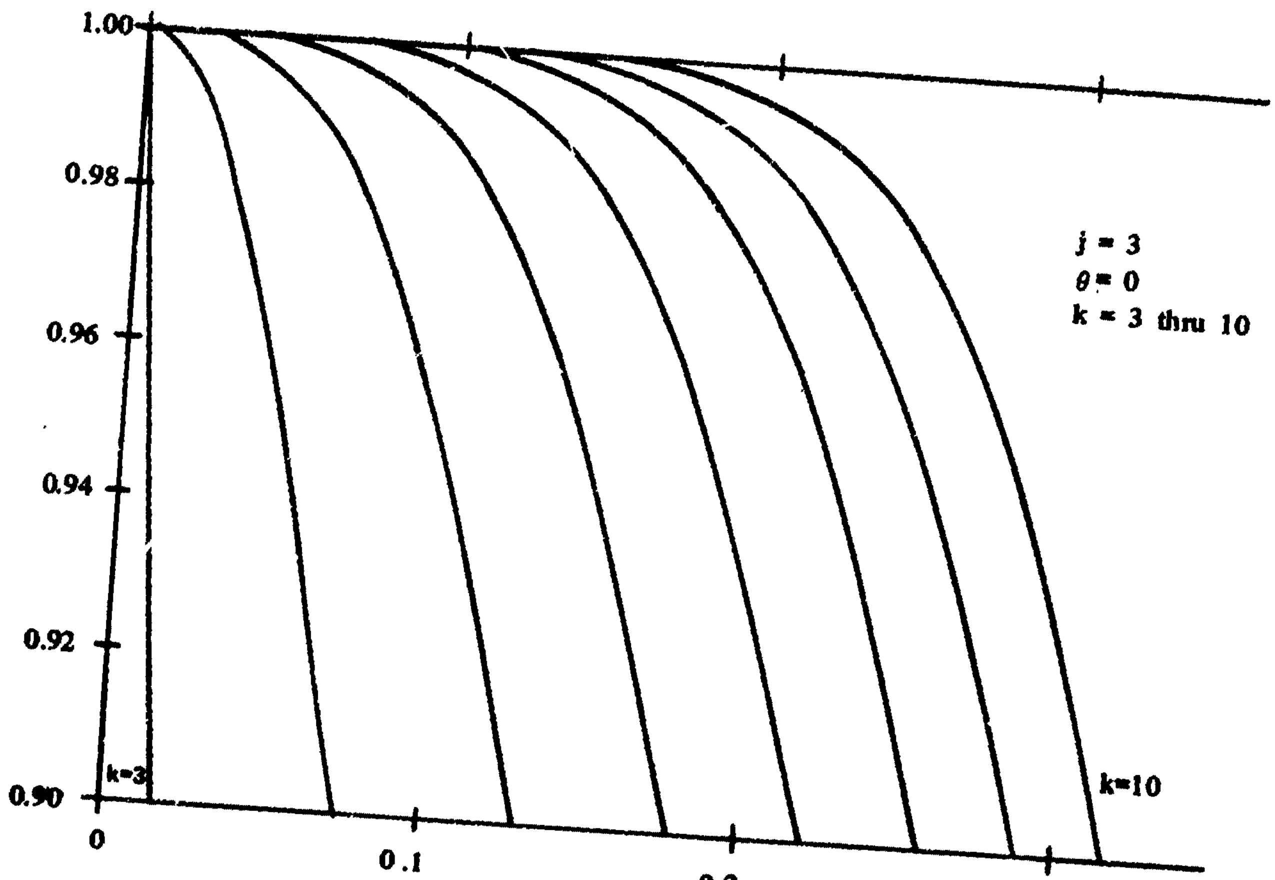

Figure 7 - Figure 4 with Scales Expanded

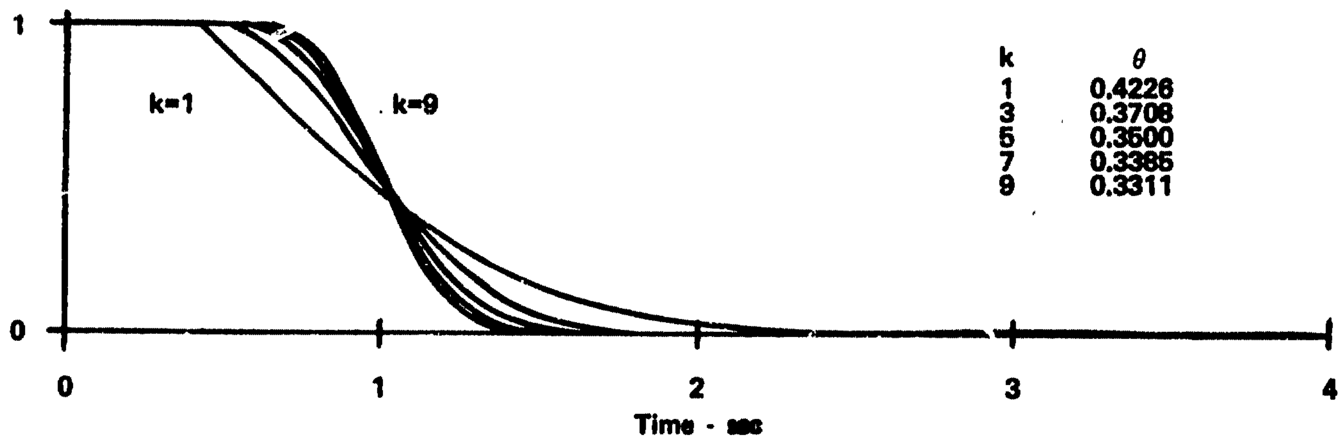

Figure 8 - Step Responses of Tape Delay Systems with j=1

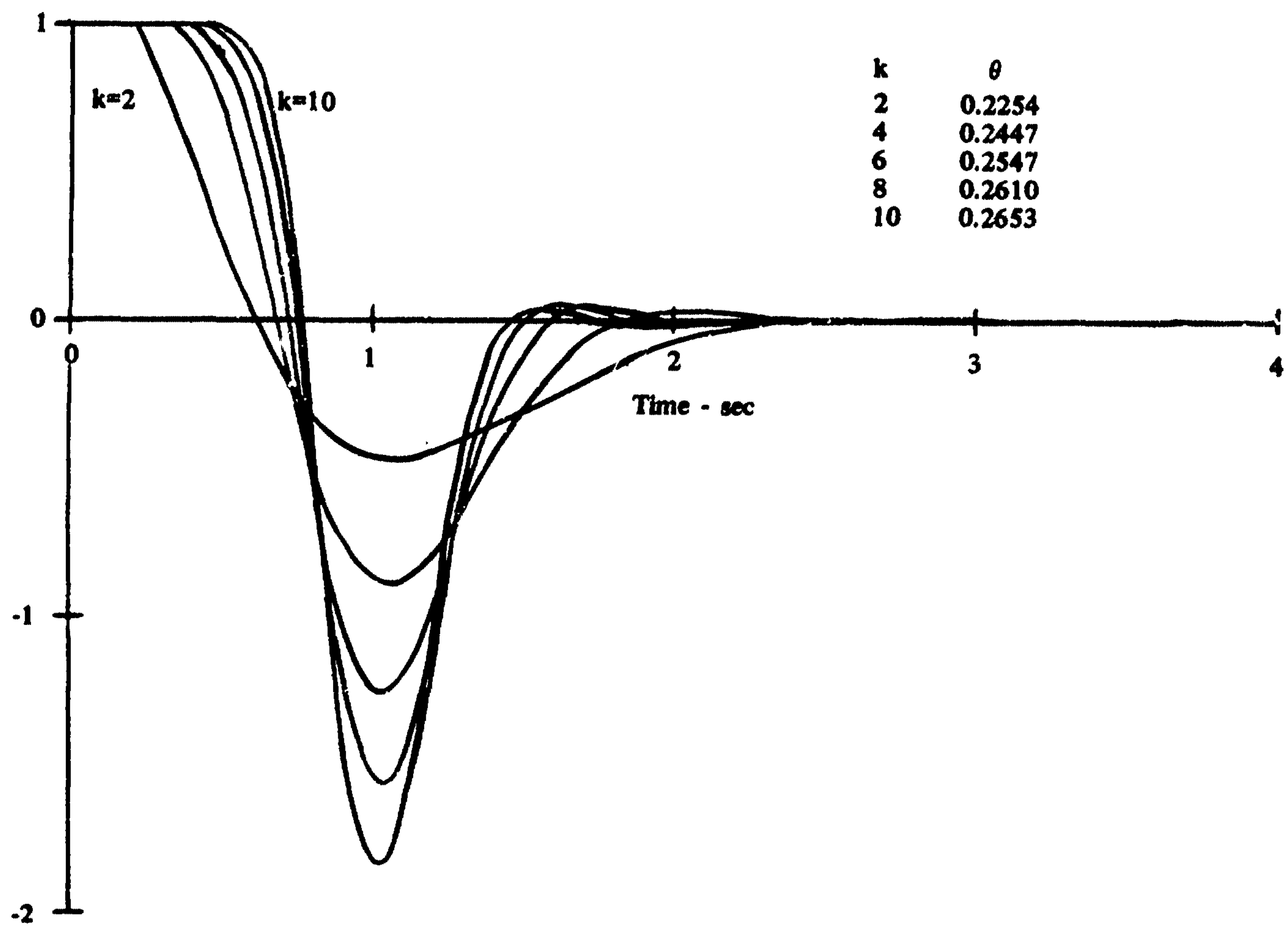

Figure 9 - Step Responses of Ta;e Delay Systems with j=2

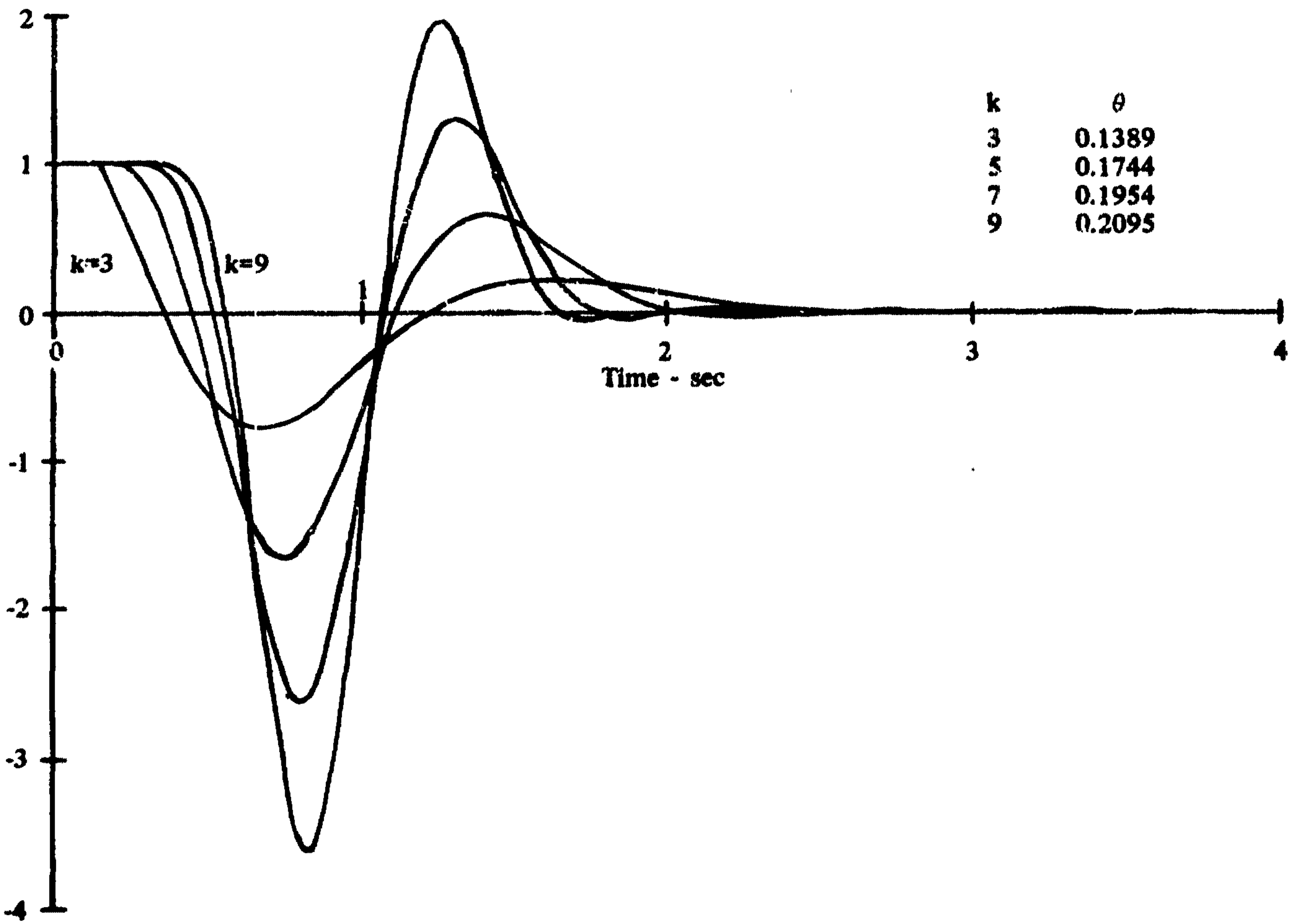

Figure 10 - Step Responses of Tape Delay Systems with j=3

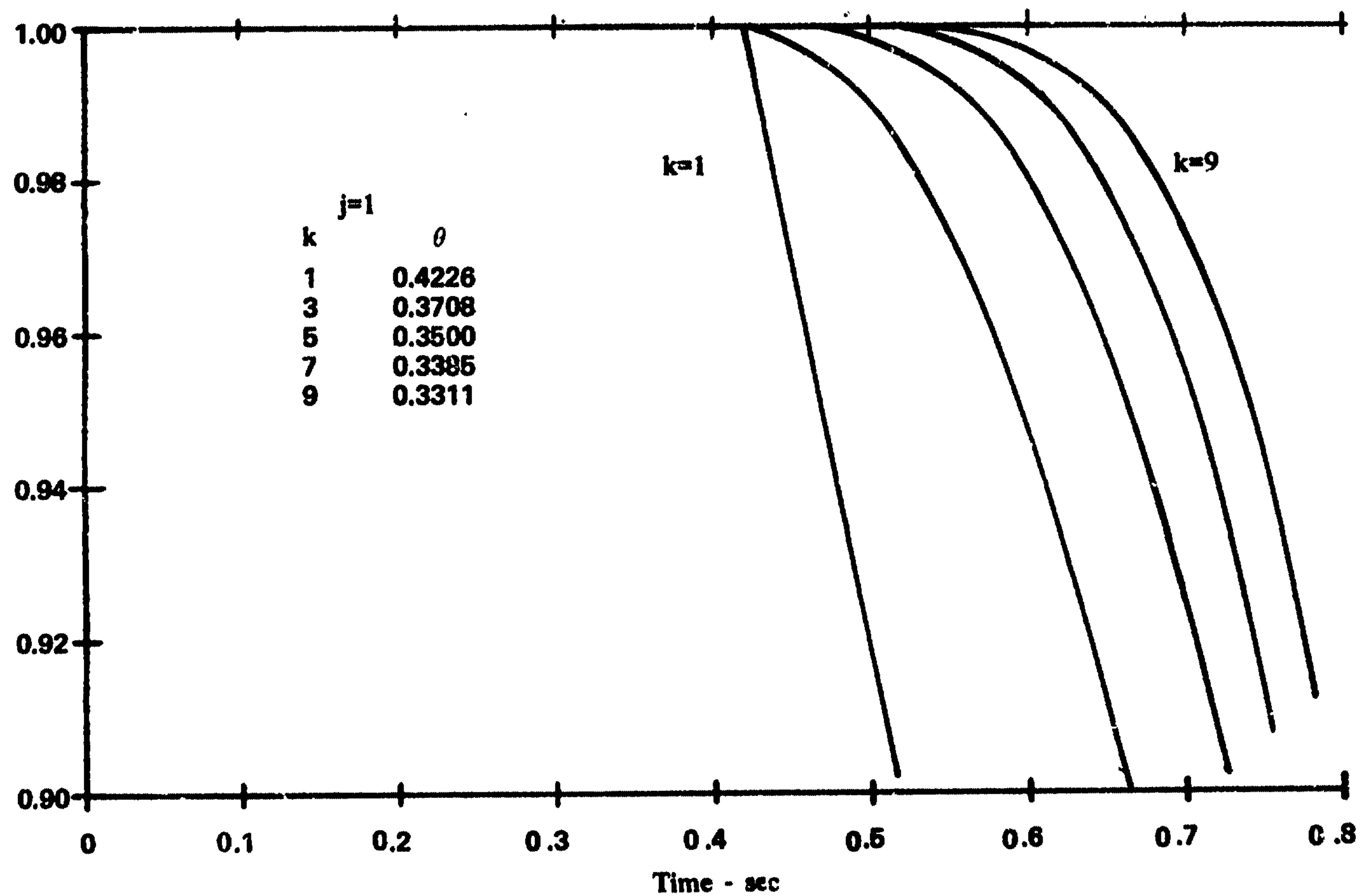

Figure 11 - Figure 8 with Scales Expanded

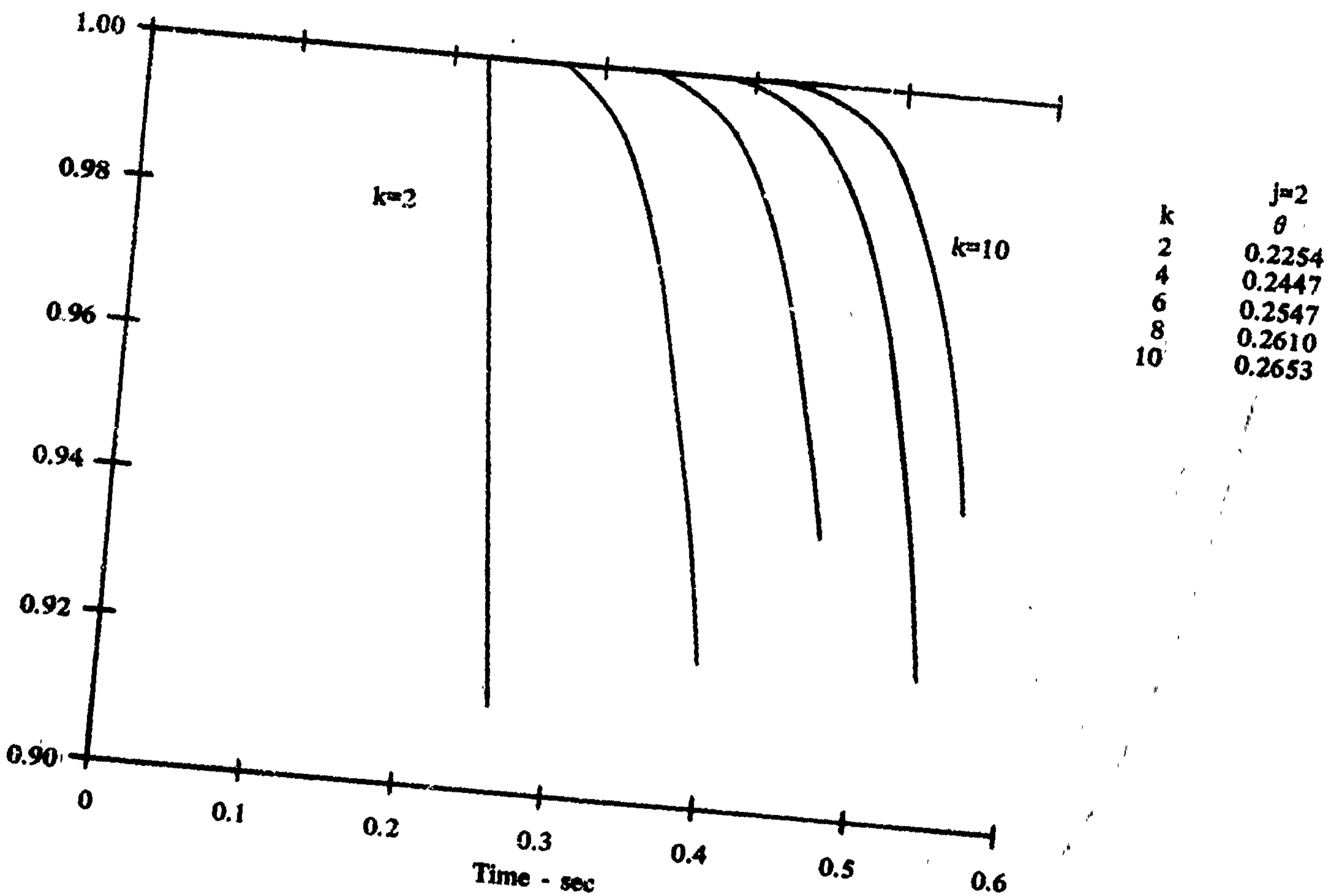

Figure 12 - Figure 9 with Scales Expanded

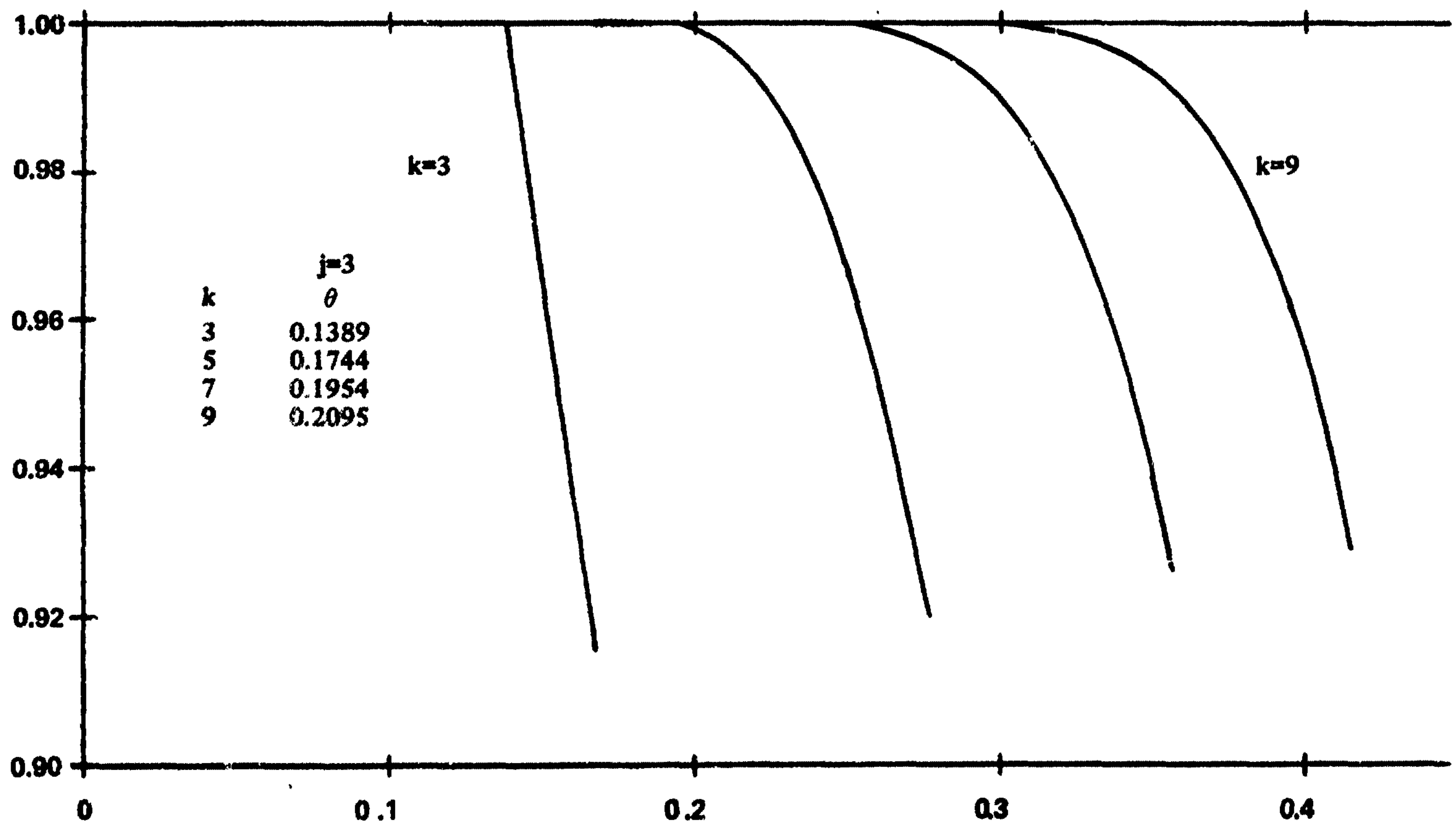

Figure 13 - Figure 10 with Scales Expanded

A comparison of the step responses shows the manner in which oscillations between the initially flat portion of step representation and the eventual zero value increase with increasing order of the system transfer-function zero at the origin. It has previously been shown that these oscillations must be present so that the first (j-1) consecutive integrals of the step response will approach zero with increasing time. The figures show that by increasing the order of the system, the duration for which the output follows the step input is increased. It is apparent from the figures that for a given value of j, improvement occurs at a decreasing rate as order is increased. However, for a given deviation from the exact step, percentage improvement between two given consecutive values of k increases with the value of j.

THE STABILIZING-SYSTEM RESONANCE

From a study of the unit-step responses, it is seen that the amplitudes of the excursions the signal takes grow with order k and order of system zero j, after the initial intervals of step transmission. A transient response of this nature implies a resonant buildup of output at certain input frequencies. The stabilizing-system output may be a steady-state signal such as that produced by water wave motion of a ship at sea. To avoid resonant distortion of the output, the stabilizing system must be designed to put the resonance region at frequencies lower than those of interest in the motion. To provide information about resonant buildup, the steady-state sinusoidal frequency responses of the MacLaurin stabilizing system for j=1, 2, and 3 are given, respectively, in Figures 14 through 16 for orders j through 10.

TIME SCALING

The duration for which the output follows the step input may be varied by time scaling the stabilizing system. Time scaling is accomplished by substituting Tp for p in the describing transfer function and simulating the result. The delay time of the resulting system is the time-scale factor T. By multiplying the time axis of the step responses by T and by dividing the angular frequency axis of the frequency responses by T, the responses of time-scaled systems are obtained,

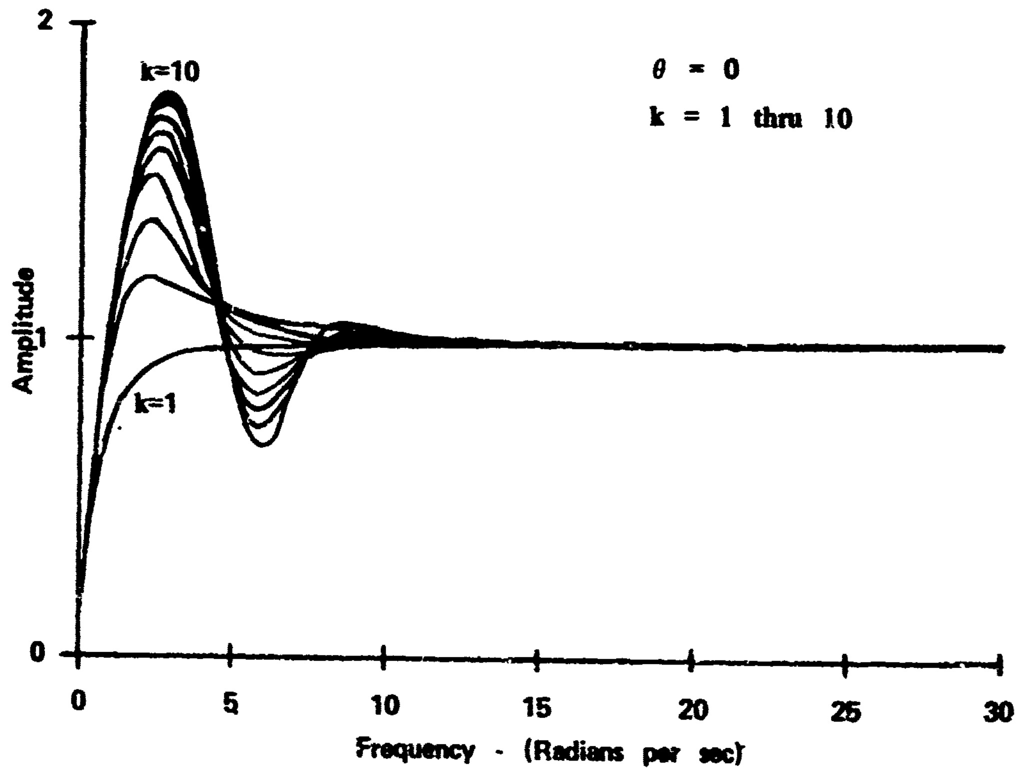

Figure 14 - Frequency Responses of MacLaurin Systems with j=1

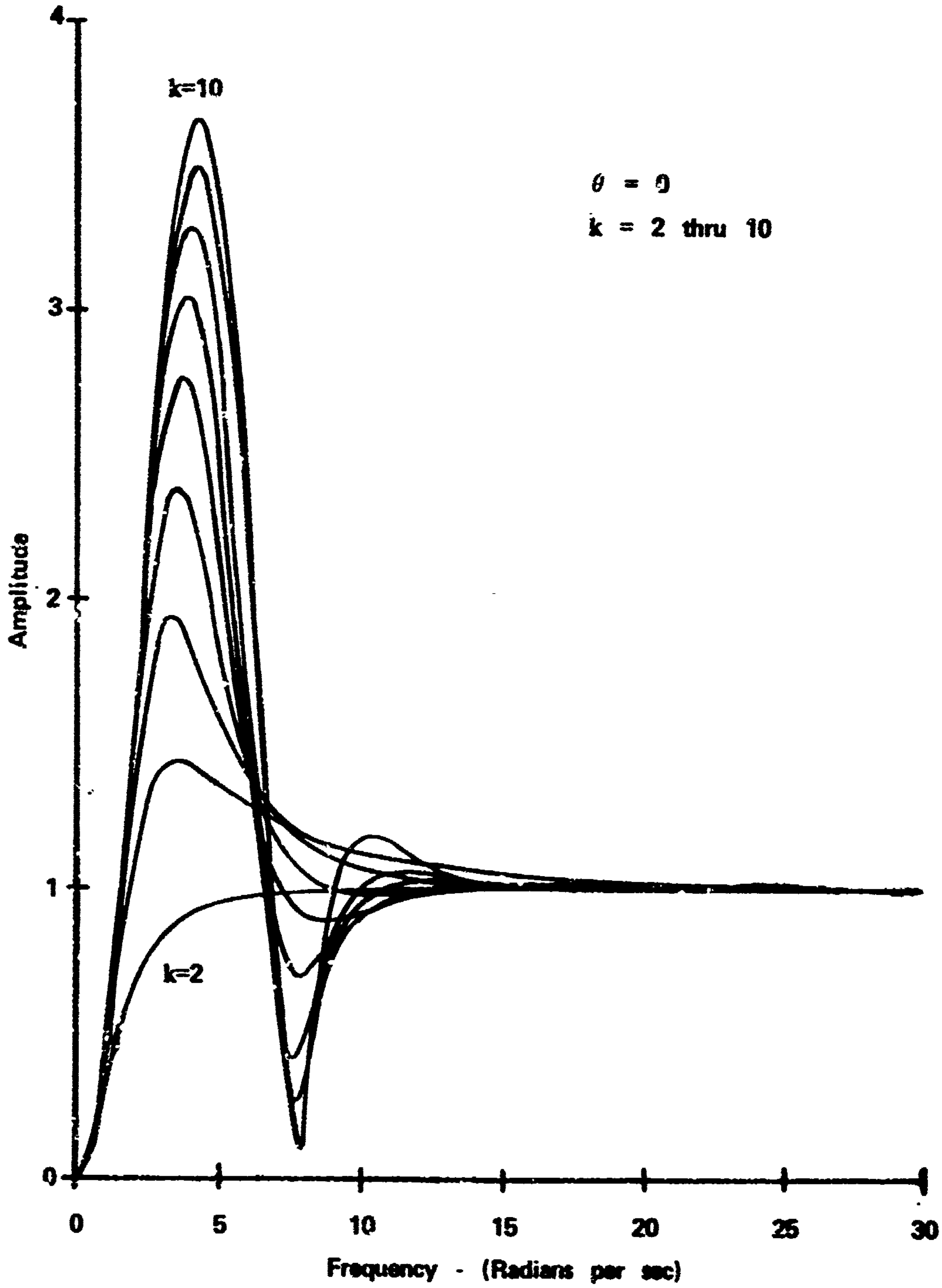

Figure 15 - Frequency Responses of MacLaurin Systems with j=2

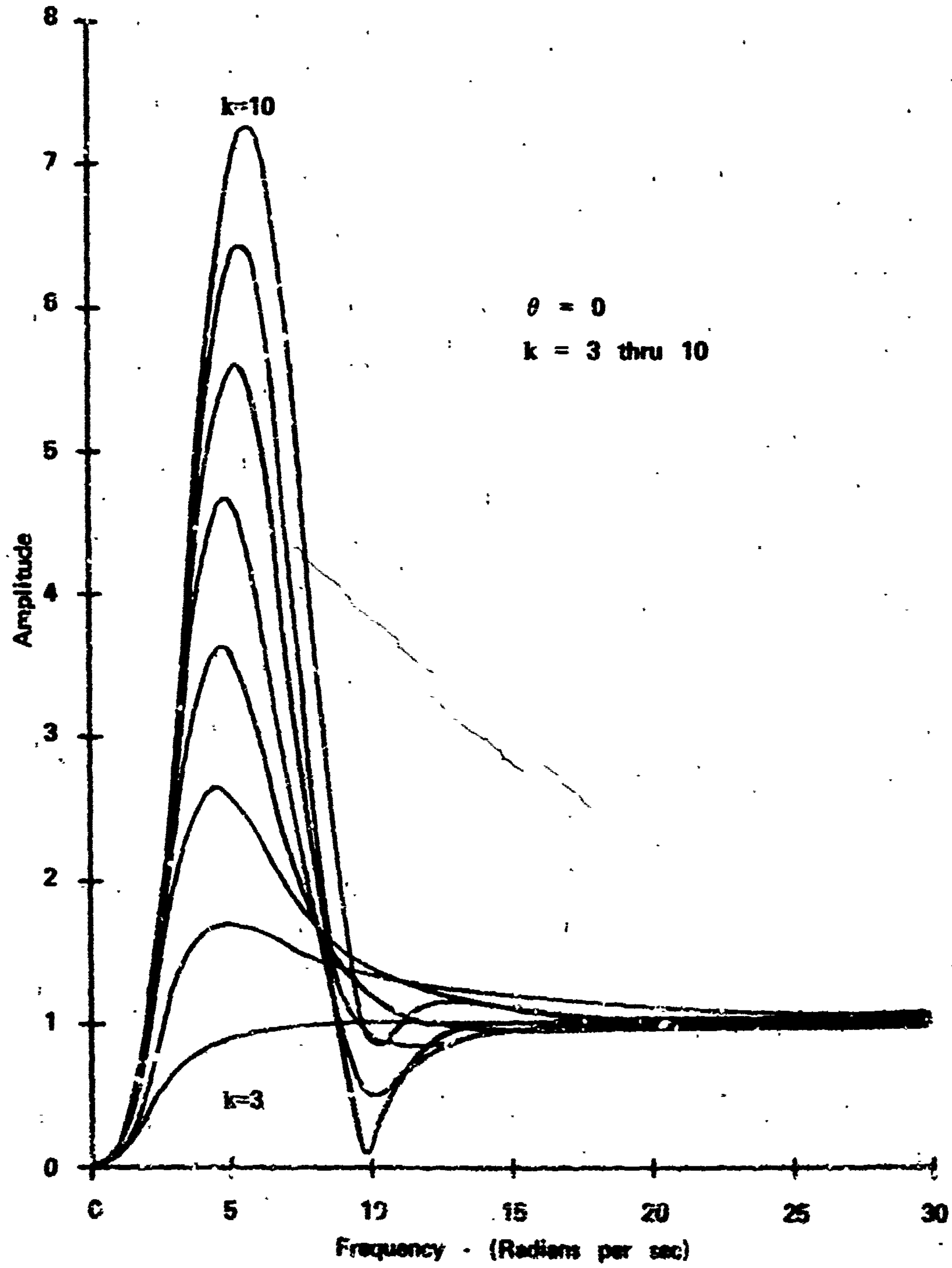

Figure 16 - Frequency Responses of MacLaurin Systems with j=3.

THE STABLE INTEGRATION SYSTEM

ANALOG SIMULATION

With reference to Equations (3) and (4), it is seen that the transfer function of the time-scaled stable integration system is

$$\frac{E_0(p)}{E_2(p)} = \frac{1}{p^i} \frac{\sum_{n=j}^{k} J_n(Tp)^n}{\exp(-\theta Tp) \sum_{n=o}^{j-1} J_n(Tp)^n + \sum_{n=j}^{k} J_n(Tp)^n} \quad (17)$$

where Tp is substituted for s in the stabilizing transfer function. Figure 17 shows the analog simulation of Equation (17) for the first three values of i and j. From Figure 17 it is seen that the value of i determines the integrator into which the input is fed, i being less than or equal to j. The value of (j-i) determines the final value of the output for a given constant, ramp, parabolic, etc, input.

When (j-i) = 0, the final value of the output for a constant input is a constant. When (j-i) = 1, the final value of the output for a constant input is zero. If the transient to be multiply integrated has an additive constant, the output of the integration system having (j-i) = 1 will be independent of this constant.

APPLICATION TO MOTION MEASUREMENT

The practicability of the stable multiple integration system may be demonstrated by using the system to obtain displacement by double integration of an accelerometer output. The accuracy of the integration system may be verified by comparing the result to a direct measurement of the displacement. Figure 18 is a picture of an accelerometer attached to a linear potentiometer and constrained to move without rotation along a guide. The displacement of the accelerometer as it is moved along the guide is measured directly by the potentiometer and indirectly by double integration of the accelerometer output using an integration system. The integration systems demonstrated will have i=2 and j=3. Since (j-i) = 1, the output is independent of accelerometer zero shifts and attitude changes taking place after the electronics start but well before the initiation of the motion.

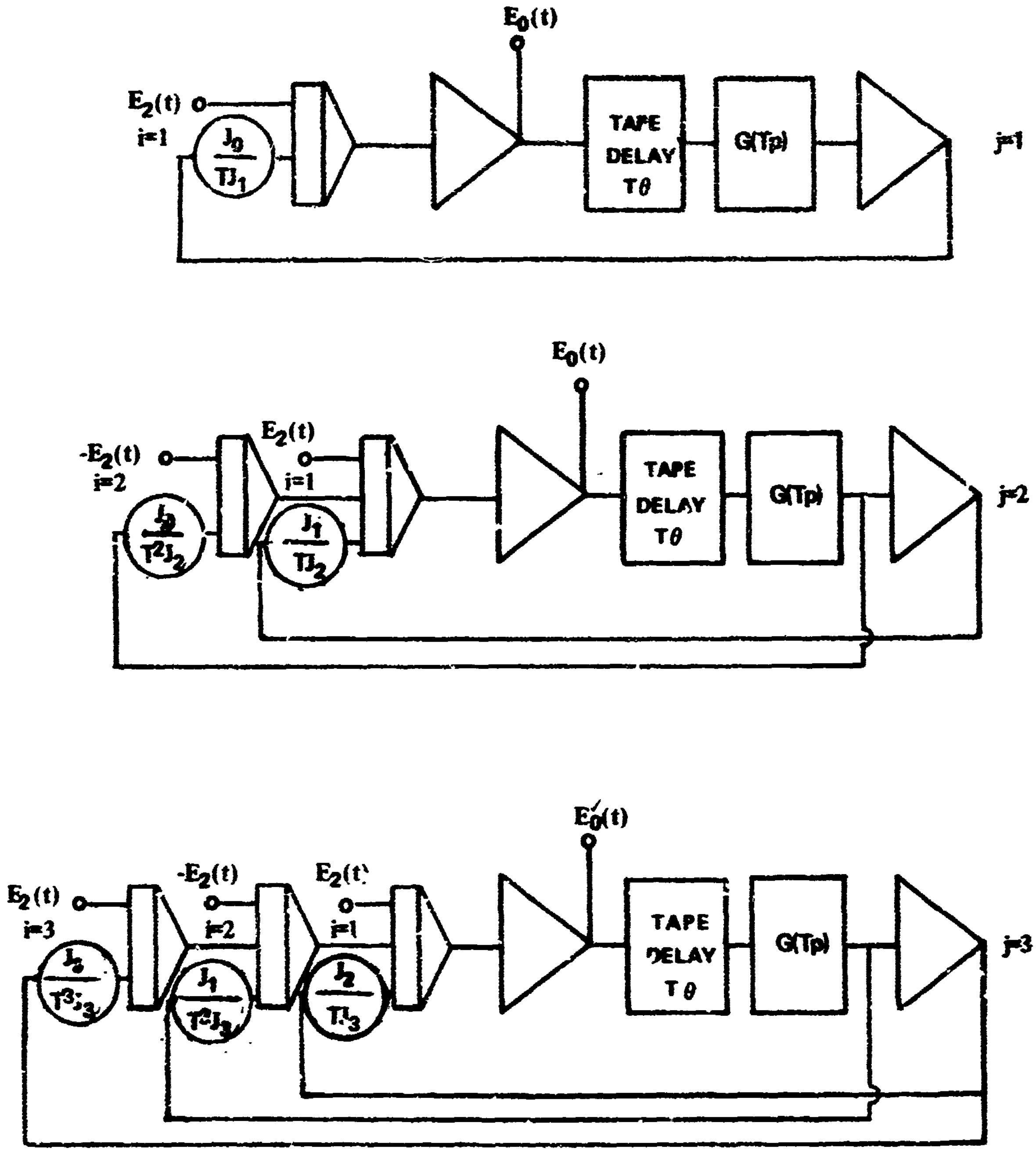

Figure 17 - Analog Simulation of the Stable Integration System

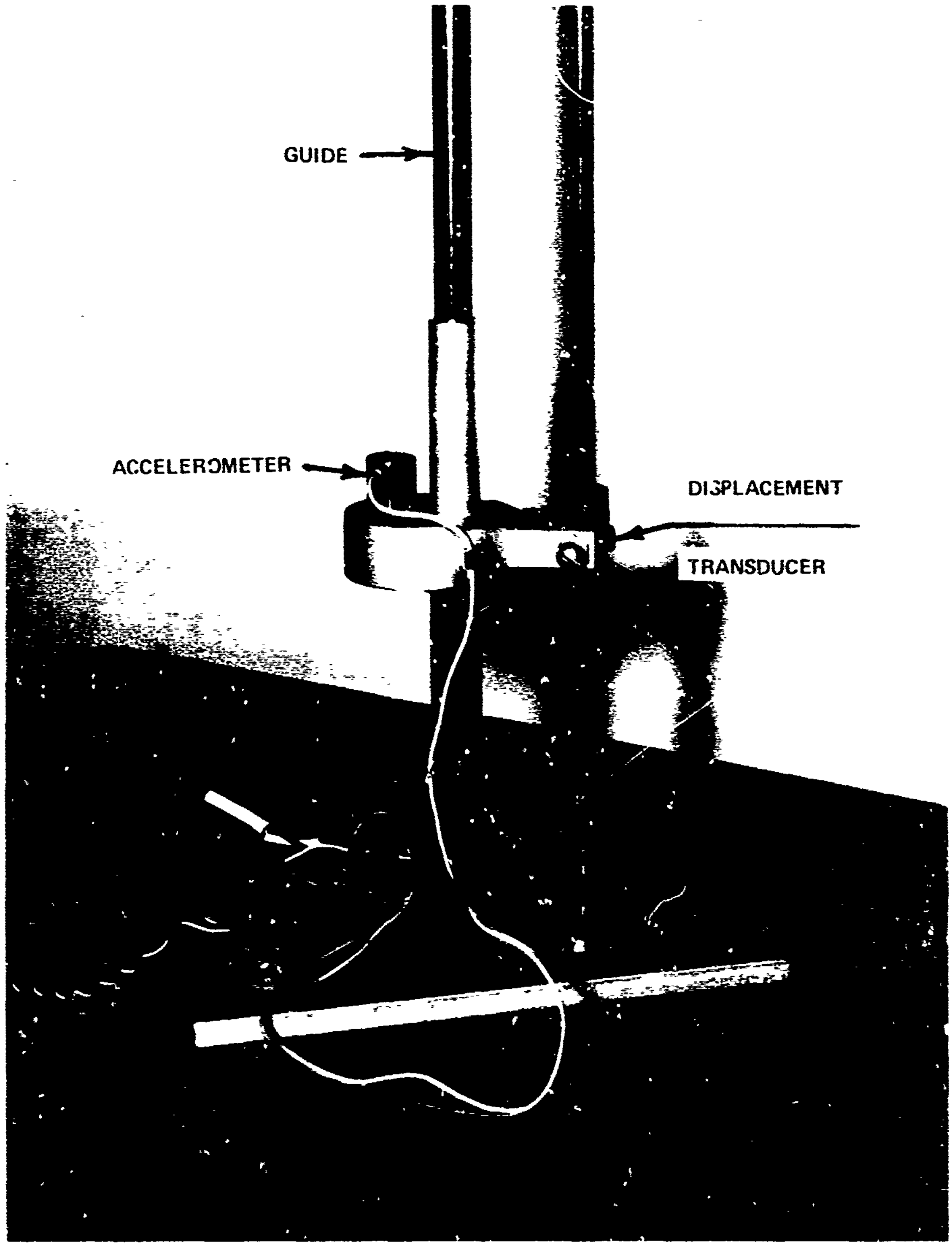

Figure 18 - Apparatus for the Direct and Indirect Measurement of Displacement

MAC LAURIN SYSTEM

Figure 19 is the analog simulation diagram of the ninth order MacLaurin system, time scaled for T=27.4. Figure 20 shows graphs of displacement obtained simultaneously from both the potentiometer and the doubly integrated accelerometer as the accelerometer is moved along the guide. A comparison of these two graphs shows the manner in which error accumulates in the doubly integrated accelerometer output when a MacLaurin system is used. Theoretically, for a MacLaurin system, error accumulates starting at the beginning of the motion. However the error accumulates very gradually and only becomes significant at later times. For a step displacement, the system being demonstrated accumulates an error of approximately 0.75 percent at 5 sec.

TAPE-DELAY SYSTEM

Figure 21 is the analog simulation diagram of the ninth order tape-delay system with θ=0.2095 sec and T=27.4. Figure 22 shows graphs of displacement obtained simultaneously from both the potentiometer and the accelerometer double integral. From these graphs it is seen that discernible error does not accumulate until about 10 sec after initiation of motion. This 10 sec of double integration is the sum of 5.74 sec of exact double integration, controlled by the tape-transport delay, and 4.26 sec of double integration with gradually increasing error, controlled by the network delay.

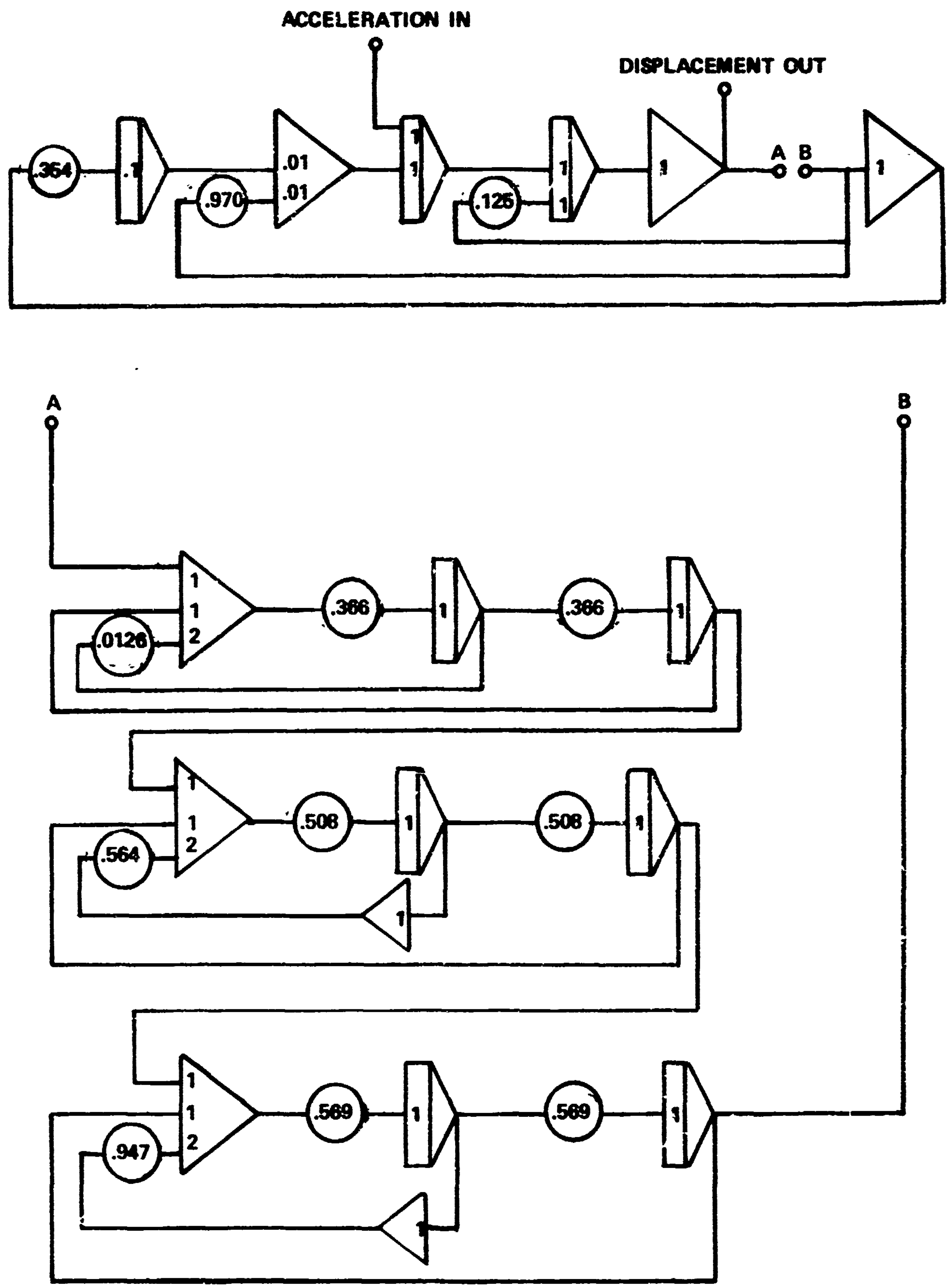

Figure 19 - The MacLaurin Double Integration System

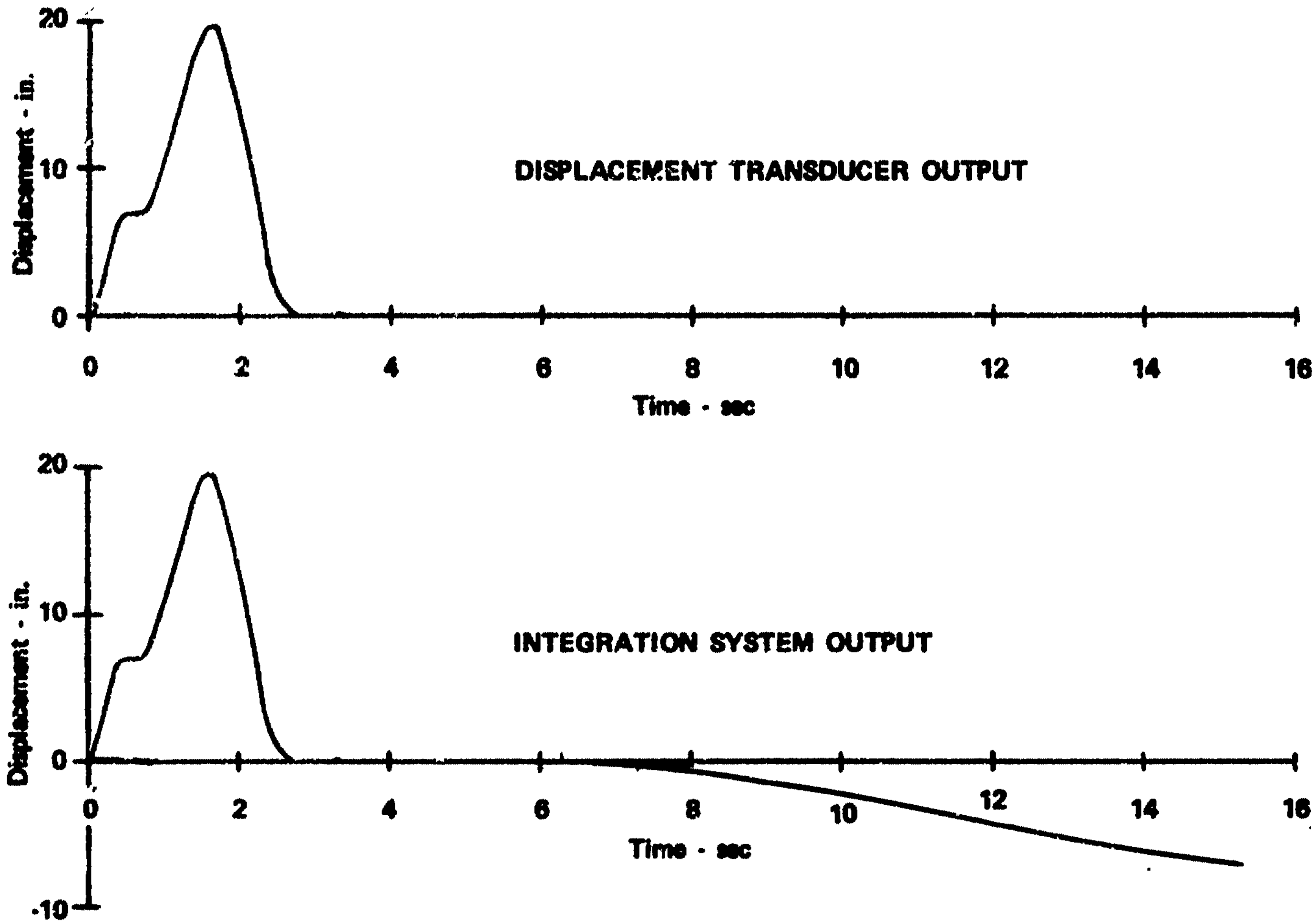

Figure 20 - Direct and Indirect Displacement Measurements, MacLaurin System

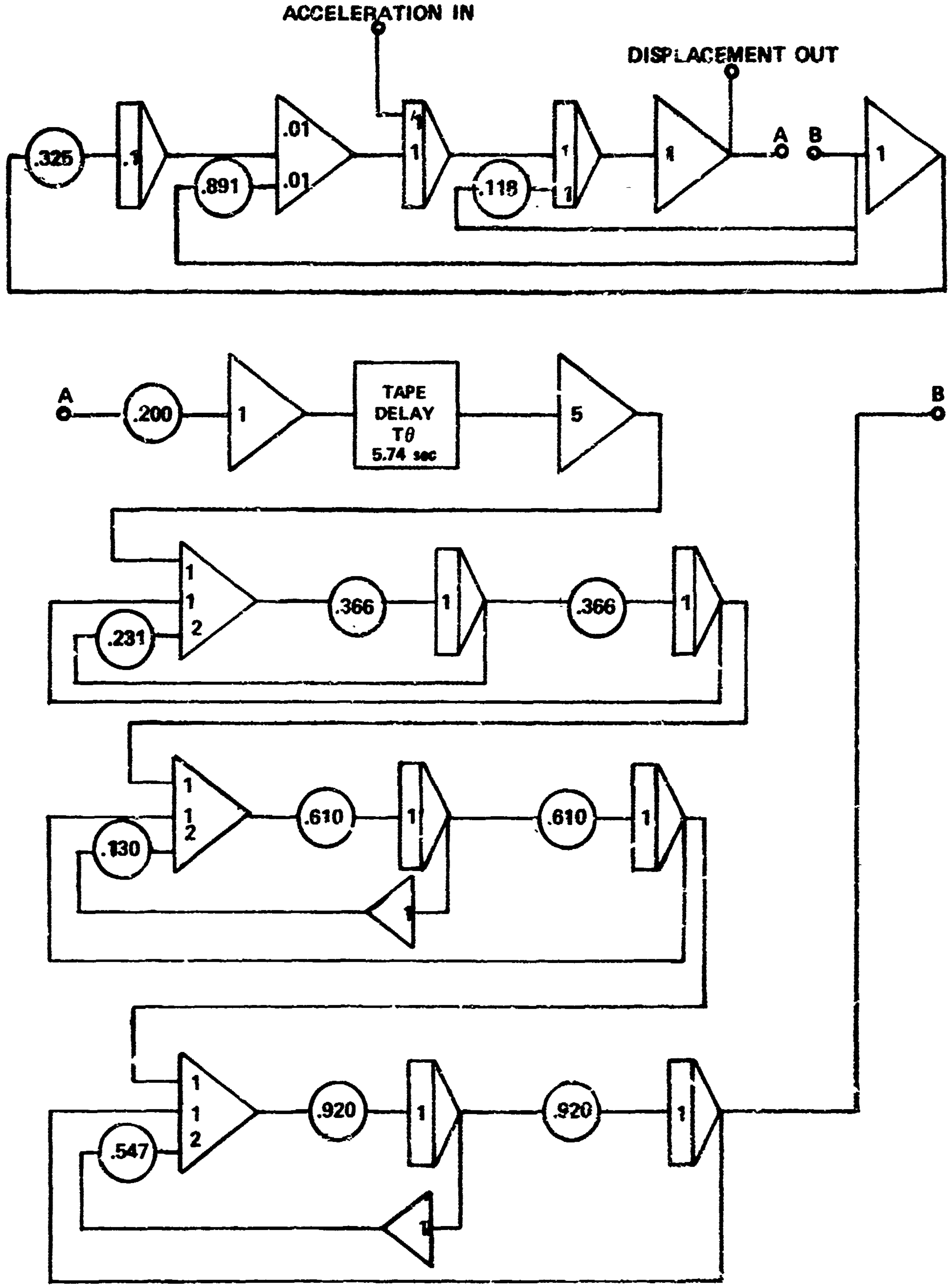

Figure 21 - The Tape Delay Double Integration System

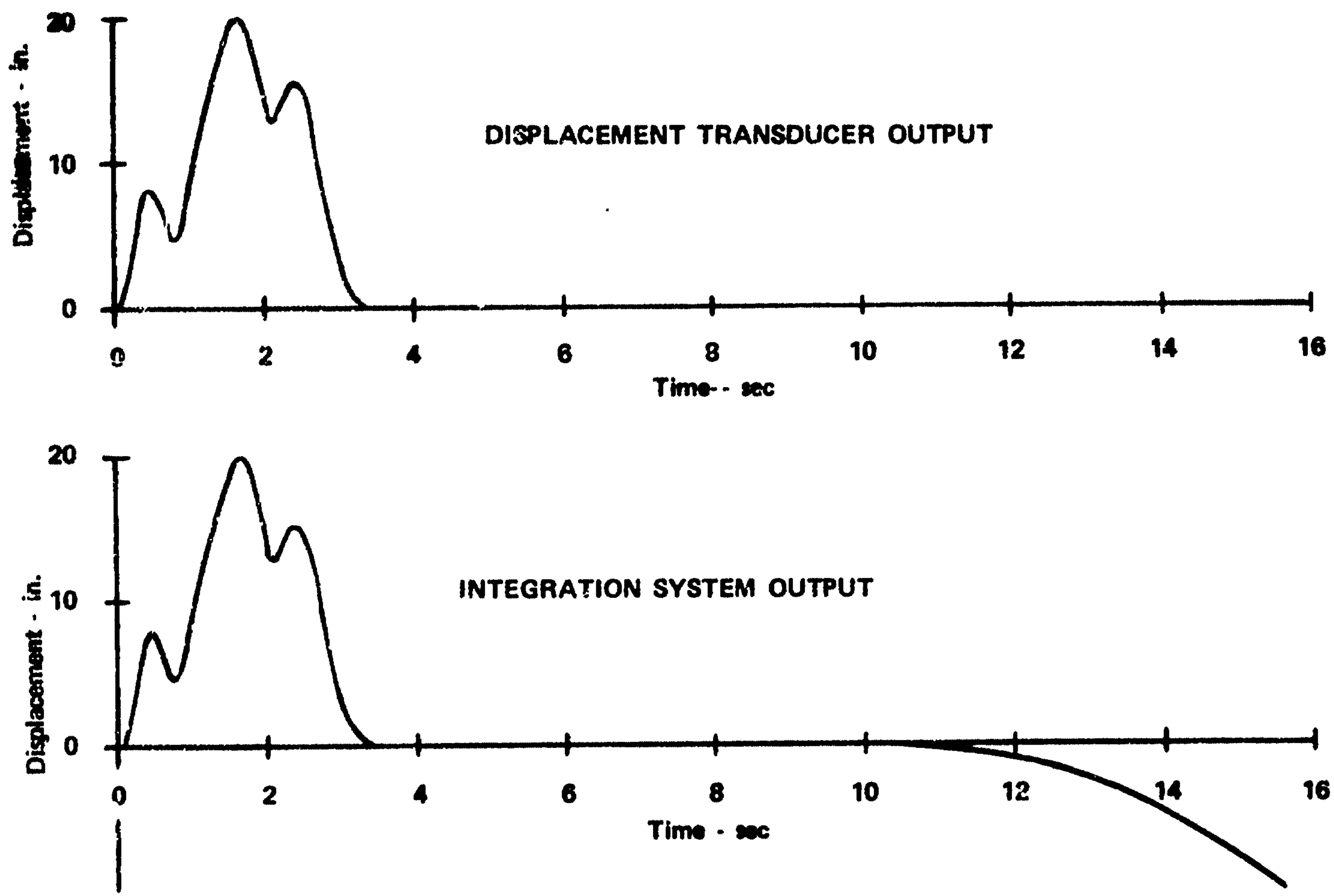

Figure 22 - Direct and Indirect Displacement Measurement, Tape-Delay System

PADE' EXPANSION OF $\exp(-\theta s)$

In the preceding systems, the physical device having the transfer function $\exp(-\theta s)$ was the tape transport. A completely electronic approximation of $\exp(-\theta s)$ may be obtained by using the Pade' expansion[4] of $\exp(-\theta s)$, having numerator and denominator polynomials of equal degree. This Pade' expansion is given by

$$\exp(-\theta s) \approx \frac{\sum_{n=o}^{q} A_{n,q}\left(-\frac{\theta}{2}s\right)^{n}}{\sum_{n=o}^{q} A_{n,q}\left(\frac{\theta}{2}s\right)^{n}} \qquad (18)$$

where q is the degree of the numerator and denominator polynomials of the expansion. and the $A_{n,q}$ coefficients are the same as appear in Equations (12). When the right-hand side of Equation (18) is given as e raised to a power series in s, q-1 consecutive odd powers of s after the first are zero as in Equation (8). Therefore the Pade' expansion of $\exp(-\theta s)$, having numerator and denominator polynomials of equal degree satisfies the criterion of delay approximation previously established.

RIPPLE SYSTEMS

When design data for tape-delay systems are used with the simulation of Equation (18) replacing the tape-transport delay, the resulting integration system has an output with oscillating error in the initial time interval. By increasing q, the amplitude of the error oscillations are reduced and may easily be made negligible. Since the output recording device is not used as part of the integration system when the simulation of the Pade' expansion replaces the tape transport, an additional feedback may be made at the input of the $\exp(-\theta s)$ simulation, resulting in an increase in the time interval of integration. A system, different from the tape-delay system already treated, results from adding the extra feedback; thus, a new unit-delay approximation must be derived, and its parameters must be determined using Equations (12).

Reference [4] - Truxal, John G., "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N.Y., (1955) P.548.

A complete list of references is given on page 51.

As an example of an integration system having ripple error, the system having the additional feedback will be treated using an output signal delay consisting of the simulation of the fourth degree Pade' expansion of exp($-\theta$s), followed by a second order network delay having poles only. Figure 23 is the analog simulation diagram of the double integration system with j=3. The unit-delay approximation analogous to Equation (6) for this system is

$$\frac{E_3(s)}{E_4(s)} = \frac{J_o}{J_o + J_1 s + J_2 s^2 + s^3\left[J_3 + \left(\dfrac{J_4 + J_5 s + s^2}{J_4}\right)\cdot \exp(\theta s)\right]}$$

where

$J_0 = 8.247$
$J_1 = 8.247$
$J_2 = 3.915$
$J_3 = .16649$
$J_4 = 129.14$
$J_5 = -.8959$ and
$\theta = .2497$

are determined from Equations (12). When a delay of θ seconds is approximated by simulation of

$$\exp(-\theta s) \approx \frac{105 - 105\left(\frac{\theta}{2}s\right) + 45\left(\frac{\theta}{2}s\right)^2 - 10\left(\frac{\theta}{2}s\right)^3 + \left(\frac{\theta}{2}s\right)^4}{105 + 105\left(\frac{\theta}{2}s\right) + 45\left(\frac{\theta}{2}s\right)^2 + 10\left(\frac{\theta}{2}s\right)^3 + \left(\frac{\theta}{2}s\right)^4}$$

the output step in the initial interval has an oscillating error of only 0.25 percent as shown in Figure 24. If a higher degree Pade' expansion for the θ delay had been simulated, the number of oscillations in the interval of step representation would have increased and their amplitude would have decreased.

Figure 25 shows the results of an accelerometer double integration test using the ripple system example shown in Figure 23 with T=27.4. From Figure 25 it is seen that ripple error is negligible for about 9 sec after the initiation of the motion. After the lapse of 9 sec, the response goes on to meet the requirements of stability in the same manner as in the previous systems discussed.

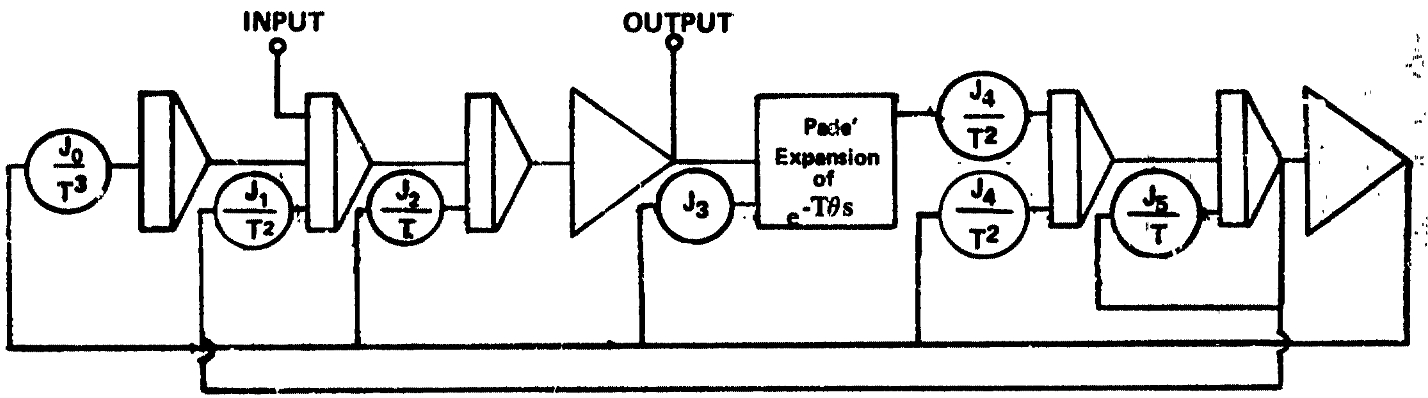

Figure 23 - Ripple Double Integration System

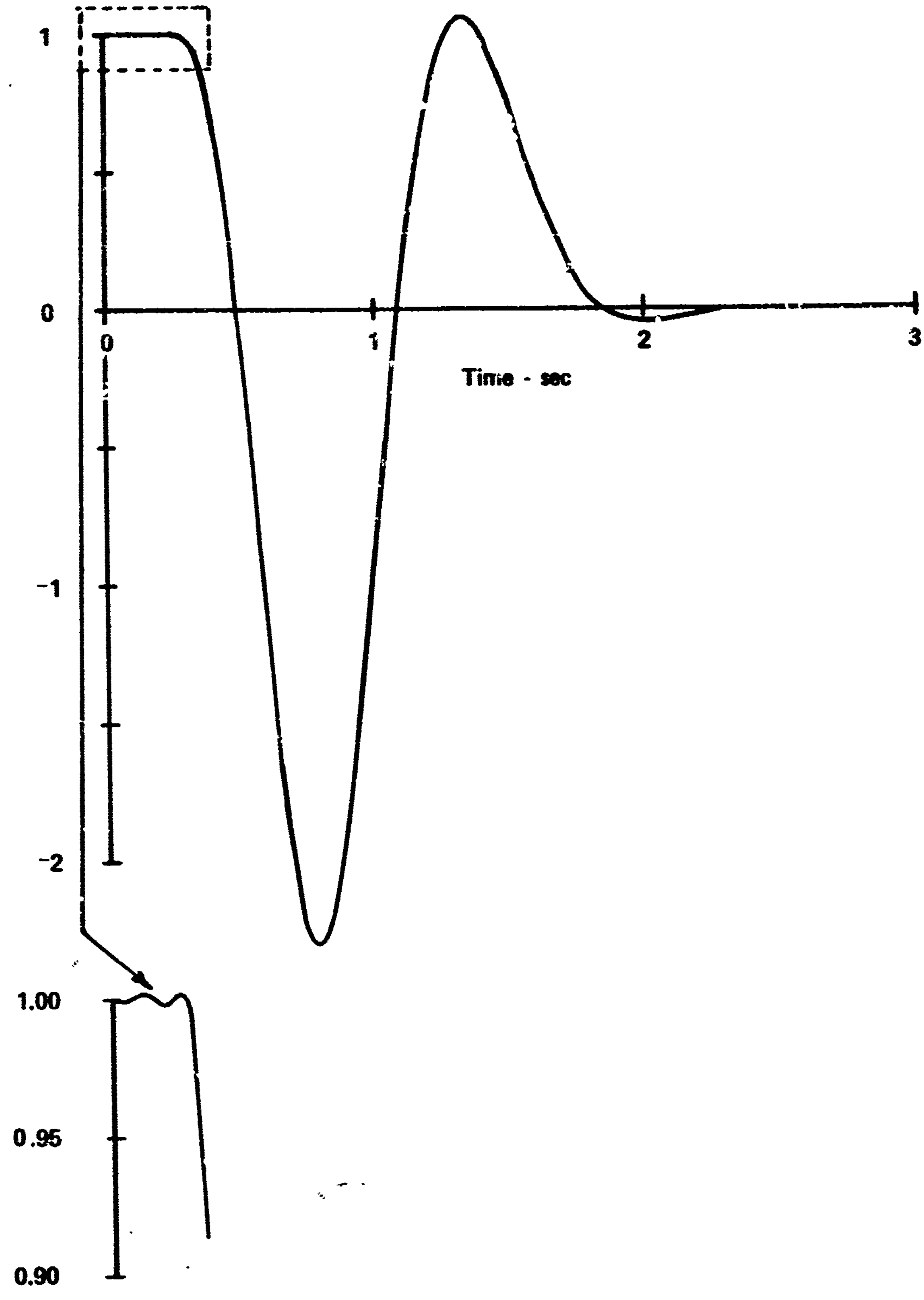

Figure 24 - Output-Step Representation, Ripple System

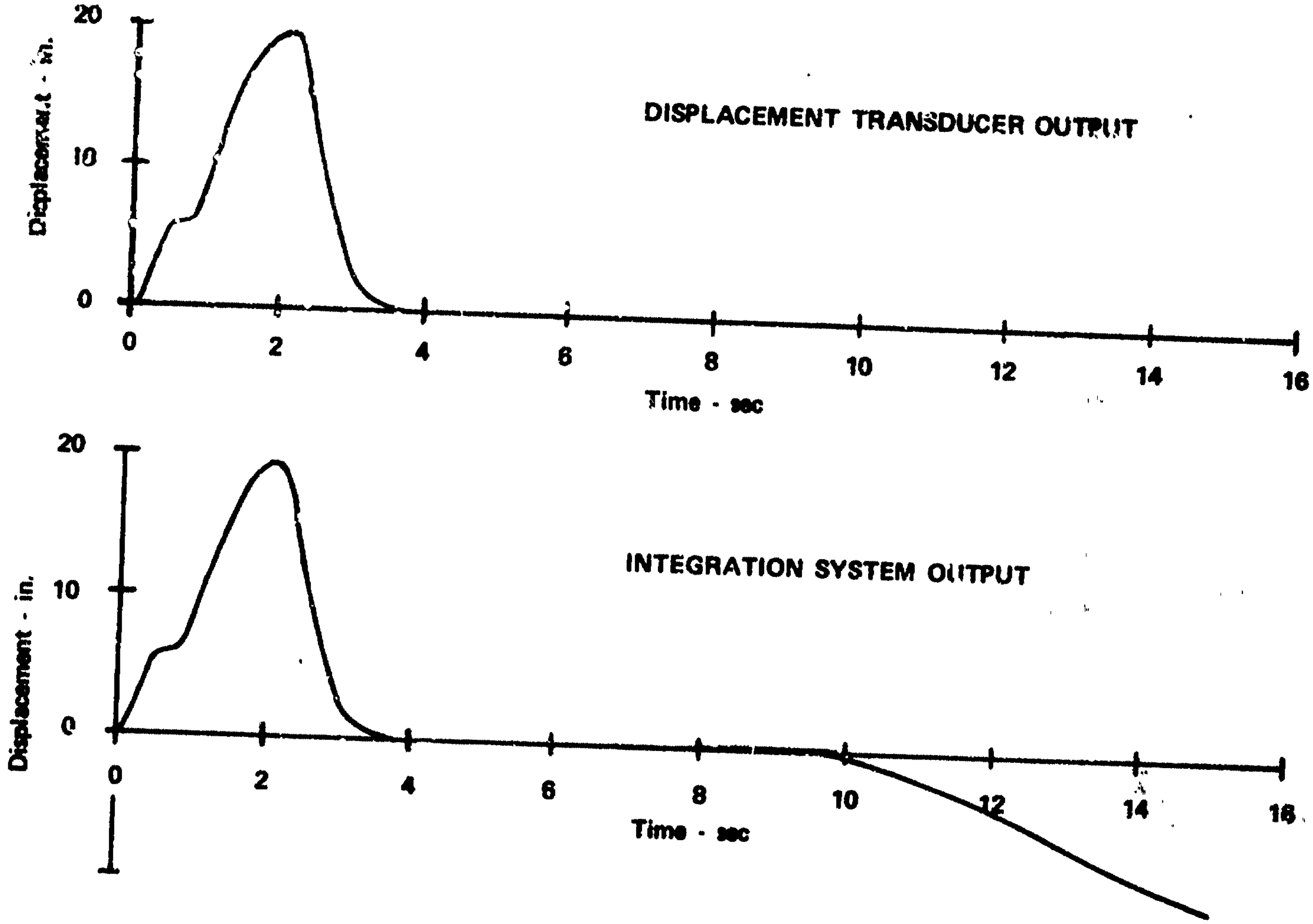

Figure 25 - Direct and Indirect Displacement Measurements, Ripple System

DISCUSSION

The success achieved in obtaining displacement by doubly integrating an accelerometer output shows that the stable integration method works, using the feedback arrangements tried. Using the tape-recorder transport delay in the integration system results in a displacement record having no error for the duration of the transport delay; however, using the network delay alone gives a satisfactory record, even though error builds up from the beginning of the motion. This buildup of error is very gradual at first and is kept within acceptable limits by raising system order and by time scaling.

When quasi-periodic transducer motion is present the integration system resonance must be considered in the system design. This consideration affects the selection of system order and time scaling and, therefore, system error. The resonance of the system is increased when the error is reduced by raising system order, and the settling time of the system is increased when the error is reduced by time scaling. No matter how error compromises are made, the requirements of stability introduce long time error.

Inclusion of a tape-transport delay in the feedback arrangement may be regarded as a mathematical design technique, assuring zero error for the duration of the delay. The actual transport delay does not have to be present. It can be approximated electronically, resulting in an integration system having oscillating error.

Three of the main factors involved in selecting a particular integration system design are quantity of electronics needed for fabrication, reliability, and system error. Of the three analyzed the tape-delay system requires the most equipment and the most maintenance. The tape transport is inherently less reliable than completely electronic devices. However, the tape-delay system has the minimum error of the systems discussed since its produces no error for the duration of the transport delay. The MacLaurin system requires the least equipment and is the most reliable. The disadvantage is that it has the smallest ratio of useful-to-settling time of the systems discussed. The ripple systems discussed are completely electronic versions of systems which could also employ a tape-transport delay. The amount of ripple error depends on the feedback arrangement as well as the quantity of electronics used to simulate the tape-transport delay. When simulation of the tape-transport delay is followed by a second order filter, the ripple error is attenuated to an acceptable level. The feedback arrangement of ripple systems

must be built to closer tolerances than are required for tape-delay and MacLaurin systems. However, the larger ratio of useful-to-settling time obtained with ripple systems may be worth the additional effort required to achieve those tolerances.

SUMMARY

Using delayed feedback to stabilize electronic integrators, allows the integrators to produce an accurate multiple integral of a transient electrical signal without it being necessary to start the integrators at the beginning of the signal. Although transient electrical signals of any origin may be integrated, using the integration method presented, the method is particularly applicable to the integration of electromechanical transducer-output signals simultaneously with their generation. The broad range of integration system design data given enables the designer of measurement systems to match the most suitable integration system with the transducer sensing the quantity whose integral is desired. The most apparent application of the integration method lies in the area of absolute motion measurement using inertial instruments. The design procedures and techniques presented herein are directly applicable to measurement systems designed to be used in explosion experiments and provide solutions to critical problems associated with designing such systems.

REFERENCES

1. Walker, Ramon R., "A Procedure for the Correction of Velocity Meter Records," David Taylor Model Basin Report 1980 (Jun 1965).

2. Gordon, John D., "Analysis and High-Frequency Correction of the Bar-Magnet Velocity Meter Response," David Taylor Model Basin Report 2187 (Apr 1966).

3. Krall, H. L. and Orrin Frink, "A New Class of Orthogonal Polynomials: The Bessel Polynomials," Transactions of the American Mathematical Society, Vol. 65, No. 1, pp. 100-107 (Jan 1949).

4. Truxal, John G., "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N.Y., (1955) p. 548